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HIGH ENERGY HEAVY ION COLLISIONS – PROBING THE EQUATION OF STATE OF HIGHLY EXCITED HADRONIC MATTER

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Abstract:

We review the recent progress in extracting the equation of state of hot dense hadronic matter from relativistic heavy ion collisions. At first a discussion of the bulk properties of infinite nuclear matter is presented. Next the theoretical approaches are developed which describe the complicated dynamics and non-equilibrium features in actual high energy nucleus–nucleus collisions: Nuclear fluid dynamics, the intranuclear cascade model, classical equation of motion simulations, the Vlasov Uehling–Uhlenbeck theory and the time dependent Dirac equation with meson field dynamics are exhibited. The recent experimental confirmation of the early hydrodynamic predictions on nuclear shock compression establishes the key mechanism for creating high nuclear density and temperatures in the laboratory, and thus the key mechanism for investigating the nuclear equation of state. Evidence for a surprisingly stiff nuclear equation of state is presented from a comparison of the distinct theoretical predictions to recent high multiplicity selected 4π data on fragment formation, pion production and collective sidewards flow. We also discuss the possible creation of a deconfined quark gluon plasma at future ultra-relativistic heavy ion facilities.

Introduction

Little is known to date about the properties of hadronic matter at finite temperatures and densities other than the nuclear ground state density $\rho_0 = 0.15\,\mathrm{fm}^{-3}$. Hadronic matter may have a rich structure in this hitherto unexplored domain of high excitation energies and compression (see fig. I.1). There are conjectures about a nuclear liquid–vapor phase transition at moderate temperatures $T < 20$ MeV and

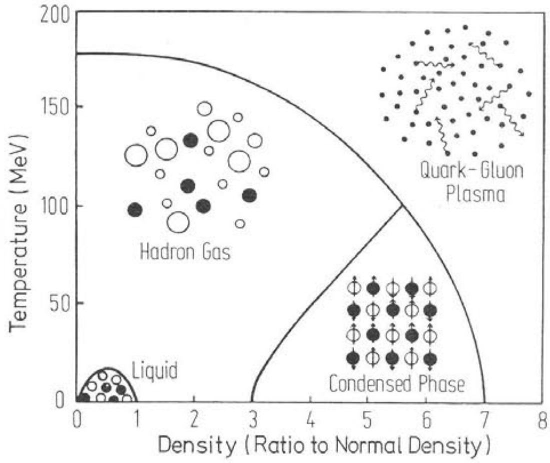

Fig. I.1. Phase diagram of nuclear matter shows the fundamentally different states that have been conjectured. Experimentally, only the point (1, 0) is known [Stö85].

densities $\rho < \rho_0$, and about abnormal nuclear matter (density isomers and pion condensates) at high densities, $\rho \approx 3–5\rho_0$. Pionization of nuclear matter is predicted for high temperatures, $T > 50$ MeV. The deconfinement phase transition from hadronic matter into the quark-gluon plasma is expected to happen at even higher densities, $\rho \approx 5–10\rho_0$, and/or very high temperatures $T \approx 150–250$ MeV. Such extreme energy densities, temperatures and baryon number densities have probably prevailed during the birth of the universe in the first fractions of a second of the big bang, and during the death of stars in supernova explosions and neutron star formation. However, until recently there was no opportunity to study these extreme conditions in the laboratory. With the recent discovery of nuclear shock compression [Gus84a, b, Ren84], which had been predicted in classic papers by Scheid and Greiner [Sch68, Sch74a, b], and extended considerably by the Frankfurt school [Ban75, Hof76, Ruc76, Stö78, 79, 80, 81, 82, 83, Buc80, 81, 83, 84, 85], the key mechanism for high compression and heating of nuclear matter in the laboratory was unambiguously established.

Experimental information about the properties of hot dense strongly interacting systems is now being sought by analyzing high energy collisions of heavy nuclei. If two colliding nuclei can stop each other, high energy densities are achievable for short time spans $t \approx 10^{-22}$ seconds. The first exploratory investigations of violent collisions between massive nuclei, $A \gtrsim 100$, have been carried out during the last few years at the Berkeley BEVALAC [Gus84a, b], i.e. in the bombarding energy range $E_{\text{Lab}} \approx$ 100 MeV/nucleon to 2 GeV/nucleon. Fundamental, exciting results have emerged, which have spurred further efforts for relativistic heavy ion facilities.

In this article we shall present a survey of the recent theoretical and experimental developments in the field of high energy heavy ion reactions. First we present the statistical concepts employed in the study of the properties of infinite hadronic systems at high density and finite temperatures, in particular the nuclear matter equation of state and the conjectured phase transitions. In the second chapter we develop various theoretical approaches to describe the dynamical evolution of the highly excited strongly interacting system in the complicated time dependent situation of an actual heavy ion collision. In the third chapter the recent 4π experiments on the production of nuclear fragments, pion multiplicities and collective flow (under shock compression) are confronted with theoretical model predictions. In stark contradiction to a decade of reluctance by some people to accept nuclear hydrodynamics [Ber75, Sob75], it is found that this model can well describe the complex processes in relativistic nucleus–nucleus collisions. We also point out the implications of these experiments for the recent attempts to determine the nuclear equation of state. It was particularly R. Stock [Sto82, 83] who followed early suggestions [Stö78, 81] to investigate the nuclear equation of state via the excitation functions of pions and extracted for the first time valuable information pointing towards a stiff nuclear equation of state. Finally, we turn to the ultrarelativistic domain, $E_{\text{Lab}} \gg 1$ GeV/N and discuss the deconfinement phase transition from hadron matter into the quark-gluon plasma, with particular emphasis on the available energy densities, the transition parameters and the space-time structure of the high energy density regions.

I. Infinite Nuclear Matter in Global Equilibrium – The Nuclear Equation of State

I.1. Statistical and thermodynamical concepts

Before we start to discuss the properties of systems composed of 'infinite nuclear matter', we have to stress that such systems exist(ed) only in astrophysical events remote in space and time – e.g. in the big bang, supernova explosions, and in the interior of neutron stars. High density matter is formed in nuclear collisions only for brief moments, and global equilibrium can not be reached at high bombarding energies. However, statistical concepts have been successfully applied to nuclear collisions, e.g. in the nuclear fluid dynamical model, which assumes that local (rather than global) equilibrium is closely approached even on rather short time scales. The assumption of statistical equilibrium in nuclear collisions can only be checked via microscopic theories which are able to describe the evolution of the system from the non-equilibrium situation to the locally equilibrated state. These theories and the questions related to the equilibration are discussed in detail in the second part of this article.

Here we want to discuss the general statistical concepts appropriate to describe the near equilibrium situation. The nuclear matter properties can be characterized by two macrocanonical variables, namely by the density ρ and by the temperature T. The discussion of the properties of a piece of hadronic matter at rest then usually starts with the definition of the energy per baryon, W, as a function of the density and the temperature. The energy per nucleon can obviously be related to the center of mass energy in nuclear collisions. It is convenient to divide the total energy per baryon $W(\rho, T)$ into a thermal and a compression part [Sch74, Bau75, Stö78–84, Buc80–85]:

$$W(\rho, T) = E_{\mathrm{T}}(\rho, T) + E_{\mathrm{C}}(\rho) + W_0 \tag{I.1}$$

where

$$E_{\mathrm{C}}(\rho) = W(\rho, T) - W(\rho_0, T=0) = W(\rho, T=0) - W_0 \tag{I.2}$$

is defined to be the compressional energy and E_{T} is the thermal excitation energy per nucleon, which is – by definition – zero if the temperature vanishes. $W_0 \approx 923$ MeV is the rest energy of a nucleon at equilibrium density. In order to understand the physical significance of $W(\rho, T)$ let us consider a piece of nuclear matter of volume V. Its energy content is given by $E_V = \int_V \rho W(\rho, T)\, \mathrm{d}V$, where $e = \rho W$ is the energy density of the matter. We would like to point out that in evaluating this quantity we have excluded the Coulomb energy and the long range part of the Yukawa energy, which lead to divergences if infinite systems are considered. Hence, in this chapter we are concerned with the short range part of the nuclear interaction only. This is the origin of the binding energy of 16 (rather than 8) MeV/nucleon used here – we are at this moment concerned with the volume term of the Bethe–Weizäcker formula only, surface and Coulomb terms are neglected. Once the functional form of W is given, standard thermodynamic relations can be used to calculate the pressure, P, entropy, S, enthalpy, H, etc. of the system at a given density and temperature. For example, the pressure is calculated from the internal energy as

$$P = \rho^2 \left.\frac{\partial E}{\partial \rho}\right|_{S=\mathrm{const}} \tag{I.3}$$

and can therefore be separated into two parts, P_{C} and P_{T}, accordingly. Similarly, one obtains the entropy of the system from the thermal energy alone: Because of Nernst's theorem, the $T = 0$ part of the equation of state does not contribute to the entropy. We will now discuss E_{C} and E_{T} in greater detail.

I.2. A model Lagrangian for a relativistic mean field theory

The total energy per baryon can, on the other hand, also be written as an expression involving kinetic and potential terms. This becomes particularly obvious in the mean field approach. The relativistic mean field theory developed by Walecka et al. [Wal74, Ser85] and Boguta et al. [Bog77, Bog83] is of special interest, since one can show that these approaches are strictly relativistically covariant, thermodynamically consistent and – as field theories – renormalizable. The relativistic mean field model discussed here consists of nucleons obeying the Dirac equation, of a classical spin zero attractive meson field (sigma) obeying the Klein–Gordon equation, of a spin one repulsive meson field (omega) obeying the Proca equation and a meson–baryon interaction between them. The resulting coupled field equations are solved simultaneously in a mean field approximation. The theory is treated in the Hartree approximation which yields an effective Lagrangian. The masses and coupling constants for the mesons are phenomenological and are adjusted to fit static nuclear matter properties.

The model Lagrangian density is

$$\mathscr{L} = -\bar{\psi}(\gamma_\mu \partial/\partial x_\mu + m_\mathrm{N})\psi - \tfrac{1}{2}(\partial\sigma/\partial x_\mu)^2 - U(\sigma) - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}m_\mathrm{v}^2\omega_\mu\omega_\mu + \mathrm{i}g_\mathrm{v}\bar{\psi}\gamma_\mu\psi\omega_\mu - g_\mathrm{s}\bar{\psi}\psi\sigma\,, \tag{I.4}$$

where

$$F_{\mu\nu} = (\partial/\partial x_\nu)\omega_\mu - (\partial/\partial x_\mu)\omega_\nu\,. \tag{I.5}$$

The potential function is taken to be a quartic polynomial in the field σ [Bog77, Bog83],

$$U(\sigma) = \tfrac{1}{2}m_\mathrm{s}^2\sigma^2 + \tfrac{1}{3}b\sigma^3 + \tfrac{1}{4}c\sigma^4\,. \tag{I.6}$$

The addition of non-linear terms to the Lagrangian (I.4) allows for a more realistic fit to other nuclear properties, such as the compressibility and effective nucleon mass.

I.3. Cold nuclear matter

For translationally and rotationally invariant infinite nuclear matter the field equations in the mean field approximation, $\sigma \to \sigma_0$, $\omega_\mu \to \mathrm{i}\delta_{\mu 0}\omega_0$ are

$$m_\mathrm{s}^2\sigma_0 + b\sigma_0^2 + c\sigma_0^3 = -g_\mathrm{s}\rho_\mathrm{s}\,, \qquad m_\mathrm{v}^2\omega_0 = g_\mathrm{v}\rho_\mathrm{v}\,, \quad \omega_i = 0\,, \tag{I.7}$$

where

$$\rho_\mathrm{v} = \frac{2}{3\pi^2}k_\mathrm{F}^3$$

is the vector density, referred to throughout the rest of the article as the baryon number density, while the scalar density is given by

$$\rho_\mathrm{s} = \frac{4}{(2\pi)^3}\int^{k_\mathrm{F}} \mathrm{d}^3k\,\frac{m^*}{(k^2 + m^{*2})^{1/2}}\,,$$

and the effective mass of the nucleon

$$m^* = m_N + g_0\sigma_0 . \tag{I.8}$$

Here we have written down the expressions for zero temperature matter; finite temperatures are discussed e.g. in [Bog81, The83]. The energy density e, pressure P and compressibility constant K at $T = 0$ are

$$\varepsilon = \tfrac{1}{2}(g_v/m_v)^2\rho_v^2 + \frac{4}{(2\pi)^3}\int_0^{k_F} d^3k(k^2 + m^{*2})^{1/2} + U(\sigma) , \tag{I.9}$$

$$P = \rho_v^2(d/d\rho_v)(e/\rho_v) , \tag{I.10}$$

$$K = 9\rho_v^2(d^2/d\rho_v^2)(e/\rho_v) , \tag{I.11}$$

respectively. The energy of a particle moving through matter with momentum k is given by

$$E = g_v\omega_0 + (k^2 + m^{*2})^{1/2} = (k^2 + m_N^2)^{1/2} + U_{eff} , \tag{I.12}$$

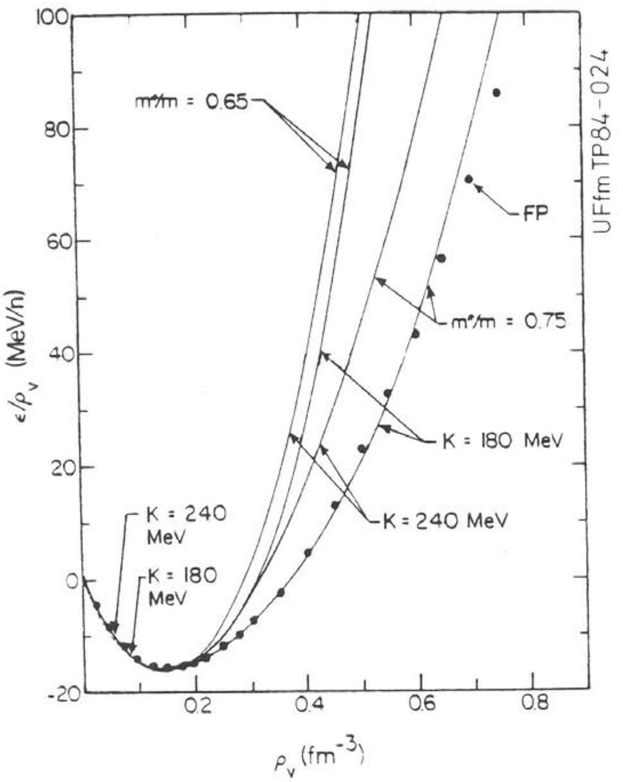

Fig. I.2. Energy per baryon of cold nuclear matter as calculated in thermodynamically consistent relativistic mean field theory [Bog83] with different parameter sets compatible with known ground-state properties of nuclei.

where $U_{\rm eff}$ is the effective nuclear potential

$$U_{\rm eff} = E - [(E - g_v\omega_0)^2 + m_N^2 - m^{*2}]^{1/2}\,. \tag{I.13}$$

This model has been applied to dynamical calculations by Cusson et al. and will be discussed in chapter III. Let us here discuss the equation of state resulting from these relativistic mean field theories for infinite nuclear matter, i.e. neglecting the space- and time-derivatives in the equations of motion, and assuming thermal equilibrium. Figure I.2 shows the compression energy, $E_C(\rho)$ of nuclear matter as calculated in the relativistic field theory [Bog83] with additional non-linear terms in the Lagrangian (solid lines) and in non-relativistic many-body calculations using the variational method (dots) [Fri81].

It is found that the results of both approaches agree for $\rho < 1.2\,\rho_0$ for any reasonable incompressibility coefficient K and effective nucleon mass m^* at saturation density ρ_0. However, at higher densities $\rho > 1.2\rho_0$ the nuclear equation of state is so sensitive to K and m^* at ρ_0 that differences of several hundred percent arise even if K and m^* are only varied within their presently assumed experimental 10–20% uncertainties. These results demonstrate that even a precise determination of the nuclear properties at normal ground-state densities does not enable us to predict the high density behavior of nuclear matter with reasonable accuracy. A theoretical determination of these properties is also very difficult in view of the fact that many body forces can play an essential role, in particular at high densities. Hence experiments which probe the dense nuclear matter directly must reinvigorate the quest for the high density equation of state of strongly interacting matter.

I.4. A phenomenological ansatz for the equation of state

Unfortunately a field theoretical treatment beyond the mean field approximation is not yet developed. To describe the collision dynamics in a time dependent theory, semiclassical approaches must be used which include the nuclear potential, but also the effects of two body collisions (see chapter II). For these approaches phenomenological equations of state have been developed. In the following we present the equation of state in such a phenomenological theory, and study further simplifying assumptions. The compression energy $E_C(\rho)$ incorporates phenomenologically the nuclear binding energy, the Fermi energy of the nucleons, hard core effects and the exchange part of the nuclear forces. It is often loosely referred to as the "nuclear equation of state". Two commonly used functional forms for $E_C(\rho)$ originate from the extended liquid drop model of Scheid and Greiner [Sch68]:

$$E_C(\rho) = K_l(\rho - \rho_0)^2/(18\rho\rho_0) \tag{I.14a}$$

$$E_C(\rho) = K_q(\rho - \rho_0)^2/(18\rho_0^2) \tag{I.14b}$$

the first being referred to as the linear- and the second as the quadratic EOS, respectively, in accord with their asymptotic increase with density.

I.5. Finite temperatures

The temperature of the system is the second thermodynamic variable of importance for the equation of state. The total energy of the system at finite temperature is being described by the interaction energy plus the kinetic energy of the particles in the system. The latter is given by interacting relativistic

Fermi–Dirac and Bose–Einstein distributions, hence the total energy per baryon is given by [Hei79, Hah85]

$$W = U + \sum_{i=1}^{\sigma_b} \left(\rho_i^0 m_i c^2/\rho + \frac{4\pi g_i}{\rho(2\pi hc)^3} \int_{m_i c^2}^{\infty} \frac{\varepsilon^2 \sqrt{\varepsilon^2 - m_i^2 c^4}}{\exp[\varepsilon/T] - 1} \mathrm{d}\,\varepsilon \right)$$

$$+ \sum_{i=\sigma_b+1}^{\sigma} \frac{4\pi g_i}{\rho(2\pi hc)^3} \int_{m_i c^2}^{\infty} \frac{\varepsilon^2 \sqrt{\varepsilon^2 - m_i^2 c^4}}{\exp[(\varepsilon + U - \mu)/T] + 1} \mathrm{d}\varepsilon \tag{I.15}$$

where the first sum runs over the Bose-degrees of freedom, the pion, the η-meson, and heavier mesons, while the second sum is over all the excited states of the nucleon — the Δ (1232) resonance being the most important resonance in the GeV/nucleon energy region. Here it is assumed that all nucleonic resonances feel the same interaction energy per particle U, which is assumed to depend only on the total baryon density ρ [Hei79, Hah85]. The potential energy must be included into the Fermi – Dirac distribution function in a selfconsistent treatment.

The baryons are assumed to be in chemical and thermal equilibrium and therefore have the same chemical potential μ. Both the chemical potential and the interaction potential for the bosons are taken to be equal to zero. ρ_i^0 is the contribution of the Bose ground state to the density of the boson phases. The connection between the baryon density and the chemical potential reads

$$\rho = \sum_{i=\sigma_b+1}^{\sigma} \frac{4\pi g_i}{(2\pi hc)^3} \int_{m_i c^2}^{\infty} \frac{\varepsilon \sqrt{\varepsilon^2 - m_i^2 c^4}}{\exp[(\varepsilon + U - \mu)/T] + 1} \mathrm{d}\varepsilon\,. \tag{I.16}$$

The number of mesons can be calculated via

$$N_i = g_i/\{\exp(m_i c^2/T) - 1\} + \frac{4\pi g_i V}{(2\pi hc)^3} \int_{m_i c^2}^{\infty} \frac{\varepsilon \sqrt{\varepsilon^2 - m_i^2 c^4}}{\exp[\varepsilon/T] - 1} \mathrm{d}\varepsilon\,. \tag{I.17}$$

We also need the connection between U and the compression energy E_C. For $T \to 0$, eq. (I.16) becomes

$$(\mu - U)^2 = m^2 c^4 + (\rho C/g)^{2/3} \tag{I.18}$$

with $C = 6\pi^2 (hc)^3$. In the same limit, we get for eq. (I.15)

$$W(T=0) = 0.75X + U + \frac{m^2 c^4}{8} \left\{ \frac{3X}{X_1^2} - \frac{3m^2 c^4}{X_1^3} \ln[(X + X_1)/mc^2] \right\} \tag{I.19}$$

where $g = 4$, $mc^2 = 939\,\mathrm{MeV}$, $X = \sqrt{m^2 c^4 + X_1^2}$, $X_1 = (\rho C/g)^{1/3}$, assuming that for $T = 0$ only the nucleonic ground state is populated, which should be true for small densities. This point can be questioned, if the nucleon–Δ interaction is much stronger than the nucleon–nucleon interaction [Bog81]. Expanding (I.18) and (I.19) for small densities, we obtain the well known approximation for the energy

$$\mu - U \approx mc^2 + \tfrac{1}{2}X_1^2/mc^2 + \ldots \tag{I.20}$$

$$W \approx U + mc^2 + 0.3(\rho/\rho_0)^{2/3}(h^2/m)(6\pi^2\rho_0/g)^{2/3} + \ldots \tag{I.21}$$

The difference between the exact expression (I.19) and the approximation (I.21), is about 1 MeV for $\rho/\rho_0 = 3$. By comparing (I.1) and (I.19) we finally get the relation between U and E_C:

$$U(\rho) = E_C + W_0 - 0.75X - \frac{m^2c^4}{8}\left\{\frac{3X}{X_1^2} - \frac{3m^2c^4}{X_1^3}\ln[(X + X_1)/mc^2]\right\}. \tag{I.22}$$

For the pressure, we have [Hei79, Hah85]

$$P = -T\sum_{i=1}^{\sigma_b}\frac{4\pi g_i}{(2\pi hc)^3}\int_{m_ic^2}^{\infty}\varepsilon\sqrt{(\varepsilon^2 - m_i^2c^4)}\ln(1 - \exp[-\varepsilon/T])\,d\varepsilon$$

$$+ T\sum_{i=\sigma_b+1}^{\sigma}\frac{4\pi g_i}{(2\pi hc)^3}\int_{m_ic^2}^{\infty}\varepsilon\sqrt{(\varepsilon^2 - m_i^2c^4)}\ln(1 + \exp[(\mu - U - \varepsilon)/T])\,d\varepsilon + \rho^2\frac{\partial U}{\partial\rho} \tag{I.23}$$

and for the entropy per baryon

$$S/N_B = P/(\rho T) - \frac{\rho}{T}\frac{\partial U}{\partial\rho} - \frac{1}{N_B}\sum_{i=1}^{\sigma_b} g_i\ln(1 - \exp[-m_ic^2/T]) + (W - \mu)/T\,. \tag{I.24}$$

I.6. Pionization in hot systems — Formation of the hadron plasma

These equations have been used in simplified models of heavy ion reactions [Hah85] to extract the temperature from pion multiplicities. The dependence of the number of pions per nucleon on the temperature as calculated with the above approach, which includes all firmly established resonances, the pion and the η meson, are shown in fig. I.3. Observe that the pion yield increases rapidly with temperature from zero to about one per nucleon at $T \approx 100$ MeV, and then flattens out — nuclear matter is gradually transformed into a hadron plasma. This becomes obvious in fig. I.4, which shows the distribution of pions over the various pion producing channels [Hah85]: At low energy, i.e. temperatures of the order of 50 MeV or less, most of the pions reside in the Bose condensed zero momentum state. At higher temperatures, the pion yield is due to nuclear resonances. The Δ-(1232) resonance is of particular importance in the BEVALAC energy regime, $E_{Lab} \approx 1$ GeV/nucleon, while the more massive resonances become important at temperatures above $T \approx 100$ MeV. Figure I.3 can be used to extract the temperatures in the moment of pion emission from the observed pion yields [Hah85]. One finds that the temperature rises smoothly with the bombarding energy, reaches about $T = 100$ MeV at the top

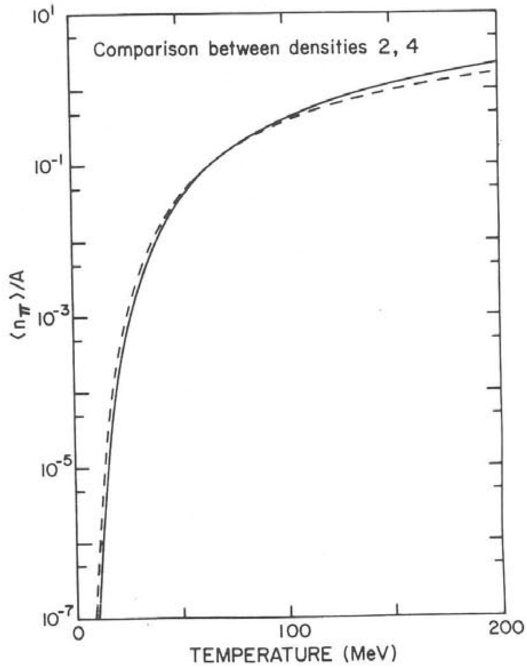

Fig. I.3. Pion multiplicities versus the temperature for baryon densities two times (solid line) and four times (dashed line) normal nuclear matter density. The curves describe the properties of a hot and dense piece of infinite nuclear matter [Hah85].

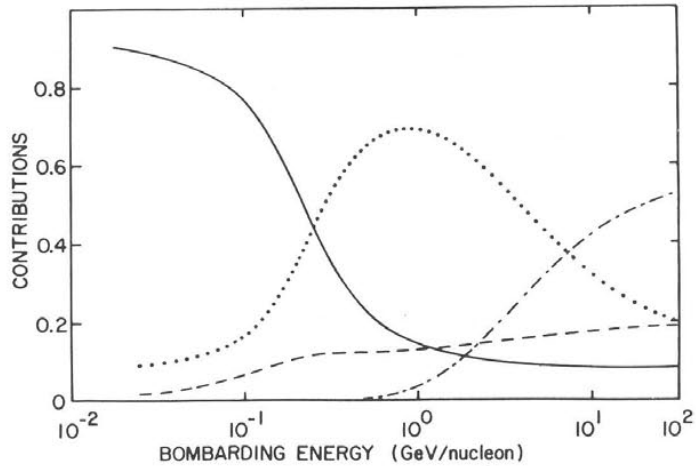

Fig. I.4. Various contributions to the pion yields for a C + C reaction. Dotted line: delta resonance; dashed line: free pion gas; solid line: Bose condensed pions; dashed-dotted: heavy resonances [Hah85].

BEVALAC energies and can be extrapolated to temperatures exceeding the critical temperature for deconfinement, $T \approx 200$ MeV, at energies in the range of relativistic heavy ion facilities presently under construction at CERN and Brookhaven, $E_{\text{Lab}} \gtrsim 10$ GeV/nucleon (see fig. I.5). This equation of state is too complicated to be of practical importance for many three dimensional model calculations. Therefore we will now discuss simpler approximations widely used in practical applications to determine the energy and density dependence of the thermal energy.

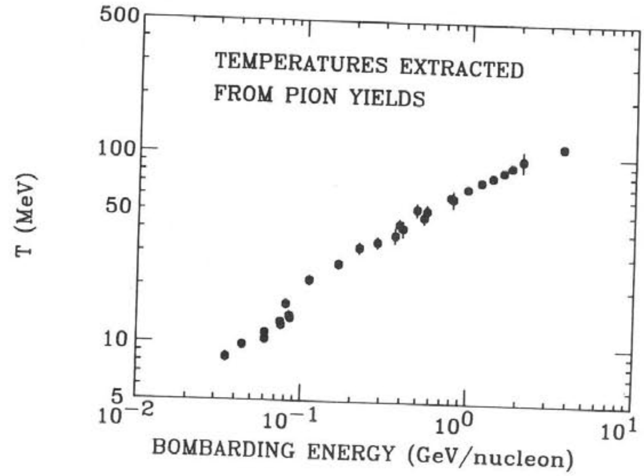

Fig. I.5. The freeze-out temperature of the pions calculated from the pion multiplicity data per nucleon. For $E_{\mathrm{Lab}} < 400$ MeV/nucleon, a freeze-out occurs for $\rho_0 < \rho < 3\rho_0$, for higher energies between two and five ρ_0 [Hah85].

The simplest ansatz for the thermal energy is the classical ideal gas $E_{\mathrm{T}} = \frac{3}{2}T$. This is actually the asymptotic value for the full non-interacting non-relativistic Fermi gas, i.e. it neglects the influence of the interactions on the thermal energy, but it contains the Fermi degeneracy energy — this means $E_{\mathrm{T}} = \frac{3}{2}T$ is the full kinetic energy, the $T = 0$ Fermi energy should be subtracted from the compressional energy if this approximation is used [San85, Hah85]. However, the classical approximation is only reasonable anyhow if the temperatures are considerably larger than the chemical potential, i.e. the Fermi energy at a given density. On the other hand, for temperatures below the Fermi energy, the Fermi gas expansion has been used:

$$E_{\mathrm{T}}(\rho, T) = \frac{\beta}{2}\rho^{-2/3}T^2 = \frac{S^2}{2\beta}\rho^{2/3} = E_{\mathrm{T}}(\rho, S) \tag{I.25}$$

where

$$T = \left.\frac{\partial W}{\partial S}\right|_{\rho} = \frac{S}{\beta}\rho^{2/3}, \qquad \beta = \left(\frac{g\pi}{6}\right)^{2/3}\frac{mc^2}{\hbar c},$$

here S being the nucleon's specific entropy.

At high temperatures, the production of resonances can be treated explicitly using the statistical approach developed above. However, a nice physical insight in the formation of resonances is obtained from the simpler classical gas ansatz for a mixture of resonances [Cha73, Hof76, Stö81]. As practically nothing is known about the N–N* and N*–N* interactions, let us assume that the N* interaction only depends on the total baryon density. Therefore the compression energy $E_{\mathrm{C}}(\rho)$ is unchanged (assuming that the subtraction of the Fermi degeneracy energy discussed above can be neglected). The first interesting quantity is the thermal excitation energy of the isobars. The thermal energy of a free ideal gas of resonances with mass $m_i c^2$ is

$$E_{\mathrm{T}_i} = \tfrac{3}{2}T. \tag{I.26}$$

The resonances can be viewed as excited nucleons (resonance pair production is not important at the above temperatures). A Boltzmann distribution for the excitation probability of the ith resonance can be assumed at temperatures much above the Fermi energy. Baryon number conservation then yields the partitions [Hof76, Sto81]

$$\lambda_i = \frac{\tau_i \exp(E_i/T)}{\sum \tau_k \exp(-E_k/T)} \tag{I.27}$$

where

$$\tau_i = \frac{g_i}{g_n} \frac{m_i^{3/2}}{m_n^{3/2}} = \frac{(2\,\mathrm{Spin}(i)+1)\cdot(2\,\mathrm{Isospin}(i)+1)}{4} \frac{m_i^{3/2}}{m_n^{3/2}} \tag{I.28}$$

is the statistical weight factor of the ith resonance, and $E_i = (m_i - m_0)c^2$ is the energy necessary for the resonance excitation.

The density of the ith phase is then given by

$$\rho_i = \lambda_i \rho \tag{I.29}$$

and the total energy density $e = \rho W$ is given as the sum over the energy densities of all phases

$$e = \sum_i e_i = \sum_i \rho_i W_i \,. \tag{I.30}$$

As all baryons are assumed to interact only via $E_C(\rho)$, the energy per resonance i is

$$W_i = m_i c^2 + E_C + E_{T_i} \tag{I.31}$$

which corresponds to a mean energy per nucleon

$$W(\rho, T) = m_0 c^2 + E_C + \sum_i \lambda_i (E_{T_i} + E_i) \tag{I.32}$$

where the mean thermal energy per baryon is

$$E_T = \sum_i \lambda_i E_{T_i} \tag{I.33}$$

and

$$\Delta M c^2 = \sum_i \lambda_i E_i \tag{I.34}$$

is the mean additional rest mass due to the occupation of the resonances with $m_i > m_0$. A free pion gas can be included via a polynomial fit [Mek78] to give the pionic energy per baryon

$$E_\pi = 1.85\rho_0(T/m_\pi)^{9/2}m_\pi\,. \tag{I.35}$$

That this is of little importance at higher temperatures has been seen above in the evaluation of the relativistic integrals, because most of the pions stem from the decay of the Δ resonance. The direct production of pions due to pion Bremsstrahlung has also been studied, but we will not discuss this here [Vas80, 84].

The pressure is evaluated from the relation

$$P = -\left(\frac{\partial E}{\partial V}\right)_S = \rho^2 \frac{\partial W(\rho, T)}{\partial \rho}\bigg|_S\,. \tag{I.36}$$

Taking nucleons only, we immediately obtain

$$P = P_C + P_T = \rho^2\,\frac{dE_C}{d\rho} + \rho^2\,\frac{\partial E_T(\rho, \sigma)}{\partial \rho}\bigg|_S\,. \tag{I.37}$$

For the compression energy (I.14a) we obtain the compression pressure

$$P_C = \frac{K_0}{18\rho_0}(\rho^2 - \rho_0^2)\,. \tag{I.38}$$

If the temperatures are small compared to the Fermi energy, the thermal pressure of the Fermi gas is given by

$$P_T = \tfrac{1}{3}\beta^{-1}S^2\rho^{5/3} = \tfrac{1}{3}\beta\rho^{1/3}T^2 \tag{I.39}$$

which leads to the relation

$$P_T = \tfrac{2}{3}\rho E_T\,. \tag{I.40}$$

This equation is valid not only for the low temperature Fermi gas limit, but in fact it holds, in the non-relativistic case, for any temperature and is also valid for a classical ideal gas as can be seen directly from $PV = NkT_k$ which is equivalent to $P = \rho \cdot T$ and with $E_T = \frac{3}{2}T$ one has $P_T = \frac{2}{3}\rho E_T$.

It is sometimes of practical advantage to use this form for the pressure if resonances are included:

$$P_T = \alpha(\rho, E_T)\,\rho\,E_T\,. \tag{I.40$'$}$$

Here the cooling influence of the reasonance is absorbed in the density and temperature dependence of α. Calculations then can be carried out with the usual form of the equation of state, eq. (I.37), supplemented with the temperature dependent α. Since a significant portion of the "thermal" energy goes into the excitation of hadronic resonances at higher bombarding energies, the relative reduction of the temperature is greatest here. This also reduces P_T, since part of the thermal energy now goes into the additional rest energy, Δm, of the resonances. The coefficient $\alpha(\rho, E_T)$ is almost independent of the density and depends strongly on E_T.

I.7. The liquid–vapour transition

At intermediate bombarding energies, $E_{\rm Lab} \approx 100$ MeV/nucleon, the temperatures are not high enough ($T < 20$ MeV) to cause substantial hadronization. However, another interesting phenomenon has been predicted to occur in the late stages of these collisions, namely when the density has dropped below normal nuclear matter density [Dan79]: The pressure diagram $P(\rho, T = \text{const})$ shown in fig. I.6 [Stö83] exhibits the maximum–minimum structure typical for matter with long range attractions and short range repulsions, i.e. a van der Waals gas. This can be interpreted as a liquid–vapor phase transition in low density nuclear matter. The nuclear equation of state exhibits a critical point at $\rho_C \approx 0.4\rho_0$ and $T_C \approx 18$ MeV. It turns out that these values are not too sensitive to the details of the assumed interaction [Kap84, Cse85]. The liquid and the vapor phase can coexist in a well determined density regime once the temperature is less than the critical T_c (the shaded area in fig. I.6). We would like to point out that moderate T values are also achieved in the late expansion stage at higher energies due to the adiabatic cooling. The Gibbs condition for thermodynamic stability of the two phase system is

$$T_{\rm liquid} = T_{\rm gas}\,,$$
$$P_{\rm liquid} = P_{\rm gas}\,, \qquad \text{(I.41)}$$
$$\mu_{\rm liquid} = \mu_{\rm gas}\,.$$

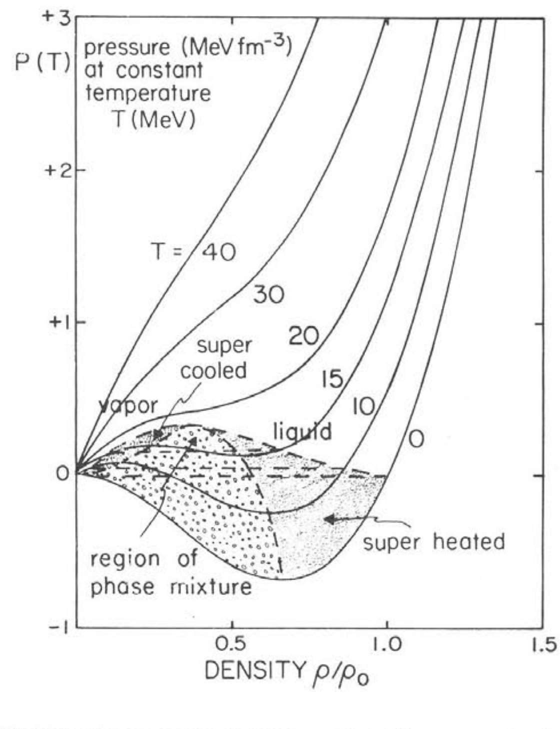

Fig. I.6. The liquid gas phase transition in nuclear matter in the pressure density plane [Stö83].

At the critical point, $P_c(\rho_c, T_c)$, the isothermal has a saddle point, while for $T > T_c$ the isothermal pressure is monotonic. Then the liquid–vapour phase separation no longer exists and this phase is therefore called the fluid phase.

The occurrence of a liquid–vapor transition in heavy ion collisions should manifest itself by substantial changes of the mass distributions of light and medium heavy fragments. We would like to point out the importance of the study of finite size effects and time scales and a detailed description of the correlations in the nuclear system in the late break-up stage of the reaction. For a detailed discussion we refer the reader to the recent review article of Csernai and Kapusta [Cse85].

I.8. Abnormal matter

The possible existence of density isomers in nuclear matter has been suggested repeatedly by many authors [Fee46, Bod71, Mig72, Lee74]. Lee and Wick observed that the non-linear scalar meson self-interaction model — the chiral sigma model — can lead to an abnormal state at high density, $\rho/\rho_0 \approx 3$–5. They found that chiral symmetry is restored in this state — i.e. the nucleons become massless. The binding energy of this state can be enormous, leading to secondary minima in the compressional energy which are several hundred MeV/nucleon deep. Another mechanism proposed to create secondary minima in $E_C(\rho)$ is the collective excitation of zero frequency spin–isospin modes in nuclear matter which carry the quantum number of the pion, therefore called pion condensation [Mig72, Wei76]. These conjectures have spurred considerable activity. However, it turns out that many of these proposals did either not attempt to describe the nuclear equation of state at other densities or else, as in the case of the linear sigma model, the description of the known properties of nuclear matter was incorrect. Since the existence of isomeric superdense matter is speculative, it is desirable to study this question in models which describe normal nuclear matter in a selfconsistent way. Figure I.7 shows a

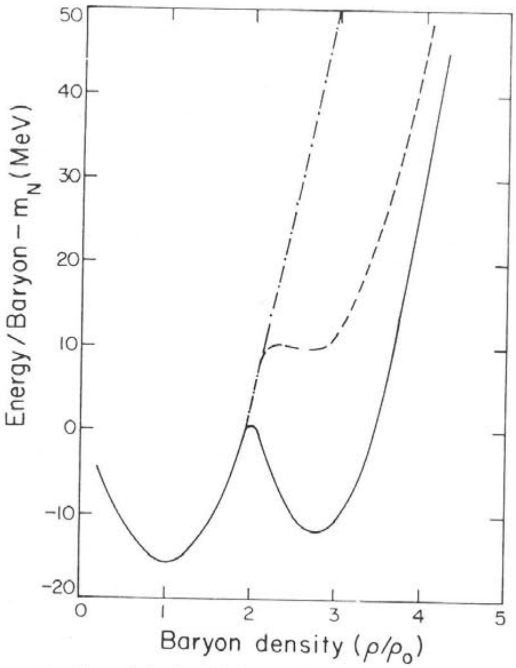

Fig. I.7. Possible abnormal states with bends or secondary minima in $E_c(\rho)$ caused by the delta resonance coupling, as predicted by the relativistic mean field theory [Bog82], which is in agreement with known nuclear properties at ground-state density.

recent calculation [Bog82] which fulfills this requirement and still predicts abnormal superdense states. The model used is the relativistic mean field theory discussed above, which is well able to describe normal nuclear matter. The abnormal state comes in by introducing the Δ-resonance into the theory. The occurrence of abnormal state depends now on the strength of the scalar interactions of the Δ. If the coupling constant for this interaction is only one third larger than the corresponding coupling of the nucleon, secondary minima occur in E_C and the abormal state is predominantly populated by the resonance rather than the nucleon. A similar mechanism has been discussed at high temperatures, leading to abundant resonance formation above a critical temperature [Hei79, Gar79]. Since the scalar coupling of the Δ is unknown, a possible existence of these baryonic resonance isomers can not be ruled out a priori. Only by doing a careful analysis of high density experiments can this question be settled.

I.9. Deconfinement and chiral transition — Creation of quark matter

A transition from the deconfined quark-gluon plasma phase to confined color singlet states has (probably) occurred during the rapid expansion of the early universe. Temperatures were very high but the net baryon charge was small. Therefore one can assume zero baryon chemical potentials in calculating the thermodynamic properties of strongly interacting matter in the early universe. It is sought to re-establish these conditions and thus enable a study of quark deconfinement in the laboratory via nuclear collisions at ultrarelativistic energies, $E_{c.m.} > 20$ GeV/N [QM79, QM80, QM82, QM83, QM84; see also the recent review Cle85]. The energy densities attainable in both the central rapidity region, i.e. the nucleus–nucleus center of momentum frame, as well as in the fragmentation regions have been estimated to be 1–2 GeV/fm^3. This range of values coincides with the energy densities at which the deconfinement transition is predicted by SU(N) Yang Mills theory (pure gluon matter) on the lattice [Cle85]. The Monte Carlo data indicate a first order phase transition at temperatures of about $T \approx 190$ MeV and zero baryon density, as seen in fig. I.8 [Eng82].

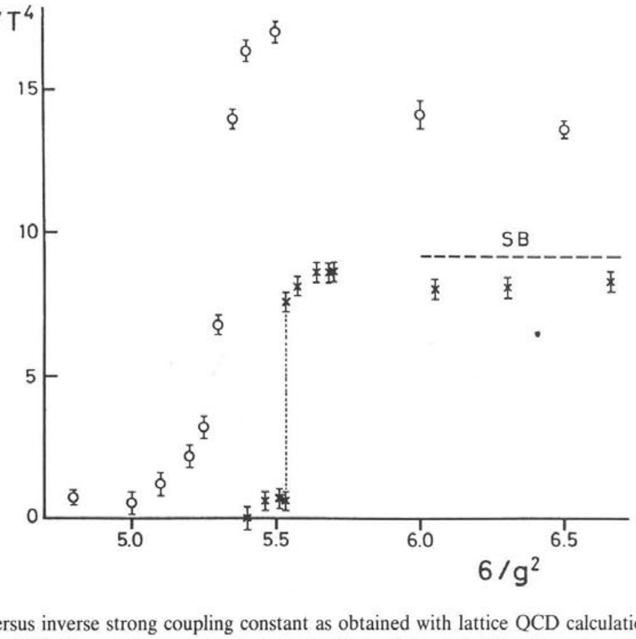

Fig. I.8. Normalized energy density versus inverse strong coupling constant as obtained with lattice QCD calculations using Monte Carlo methods [Eng82]. The crosses are the pure Yang Mills theory, the open circles include effects of dynamical fermions. A rapid change to a deconfined state is predicted at a temperature of about 200 MeV for zero chemical potential.

Unfortunately to date there is only very limited information available about the high density (high chemical potential) region. Lattice QCD calculations of the thermodynamic properties of a plasma with light quarks included are hampered by severe theoretical difficulties: The introduction of fermions on the lattice is at this time only feasible in the quenched approximation, i.e. quarks have to acquire a large mass $m_q \gg T_c$ so that the hopping parameter $(1/m_q)$ expansion converges [Cle85]. Detailed non-perturbative calculations for the situation expected to occur in violent nuclear collisions, a plasma of light quarks and antiquarks plus gluons, can therefore not be studied to date. Furthermore, inclusion of fermions requires that the charge and baryon number assume integer values for color singlet states. These problems have been studied but so far without success.

The behavior of the confined phase, i.e. hadron matter, can be described by the effective relativistic field theory of strongly interacting matter discussed in section I.2. This approach has been applied successfully to describe known properties of nuclei and nuclear matter. Though developed for normal nuclear systems, this theory may turn out very useful for a phenomenological approach to the phase transition [The83]: A sharp rise is observed for zero chemical potential in the normalized energy density e/T^4. A phase transition occurs at a critical temperature $T_c = 190$ MeV, with quite similar thermodynamic appearance as the one observed for SU(2) and SU(3) Yang Mills theory on the lattice; the order of the phase transition depends on the strength of the coupling constants [The83]. Furthermore, chiral symmetry is effectively restored in this theory just above the critical temperature. The theory does not incorporate deconfinement, though.

Hence, a different approach is necessary if one wants to study deconfinement and the quark-gluon plasma phase at least qualitatively. One can approach the transition region from high temperatures, making use of perturbative QCD to estimate the thermodynamic properties of a plasma of light quarks and gluons at finite chemical potential μ and temperature T. We would like to emphasize that sizable non-perturbative corrections can be done, but the results should still be taken only as what they are intended to be, namely a qualitative handle on the unsolved non-perturbative treatment [Stö84, Cle85].

For zero temperature, the thermodynamical potential can be written as a perturbative expansion P_{pert} in α with terms up to order $\alpha^2 \ln\alpha$, a vacuum pressure contribution Λ_{vac} (the Bag constant B in the MIT model [Cho74]) and an instanton term P_{inst} which takes non-perturbative effects partially into account [Shu80]:

$$P = P_{\text{pert}} + P_{\text{inst}} - \Lambda_{\text{vac}} ,$$

$$P_{\text{pert}} = n_f \frac{\mu^4}{4\pi^2} \left[1 - \frac{2\alpha}{\pi} - \frac{\alpha^2}{\pi^2} n_f \ln(\alpha n_f) - 0.74 n_f + 7.78 \right] , \tag{I.42}$$

$$P_{\text{inst}} = \tfrac{8}{3} C \Lambda_{\text{vac}} n_B^{-5/3} ,$$

where n_f and n_B are the number of quark flavors and baryon density, respectively and $C \approx 1000$ MeV fm^5. The dilute instanton gas term increases the pressure substantially, which results in a large decrease in the energy per baryon. In fact, the energy per baryon of the quark phase falls below the nucleon mass for a wide range of densities. Therefore this term is often omitted from the calculations.

The thermodynamical potential of a finite temperature plasma at non-zero chemical potential has been calculated up to third order in $g = (4\pi\alpha)^{1/2}$ [Kap79]:

$$-\Omega_V = P = \frac{8\pi^2}{45}T^4 + \frac{7\pi^2}{60}n_f T^4 + n_f\Big(\frac{1}{4\pi^2}\mu^4 + \tfrac{1}{2}T^2\mu^2\Big) - g^2\Big(\frac{T^4}{6} + \frac{5n_f T^4}{72} + \tfrac{1}{8}n_f\Big(\frac{\mu^4}{\pi^4} + \frac{2\mu^2 T^2}{\pi^2}\Big)\Big)$$

$$+\frac{2}{3}\frac{g^3}{\pi^4}T\Big(\pi^2T^2 + \frac{1}{2}\sum_f \int_0^\infty \frac{dp}{E_p} n_p(p^2 + E_p^2)\Big)^{3/2} . \qquad \text{(I.43)}$$

The g^3 term in this expansion corresponds to the plasmon term in quantum electrodynamics. For small temperatures, it does not converge to the zero temperature perturbation expansion. On the contrary, this term contributes a finite entropy

$$S(T=0)/V \sim \partial\Omega/\partial T = \text{const}\cdot\mu^2 \qquad \text{(I.44)}$$

to the system even at zero temperature. Furthermore, its contribution to the energy per nucleon, which is zero (as it should be) for zero temperature, is large and negative for finite temperatures. In fact, the excitation energy per baryon is decreasing with increasing temperature. Because of this unphysical behavior the plasmon term must be omitted from further calculations.

Following renormalization group arguments, the running coupling constant α can be written as

$$\alpha = \frac{4\pi}{11 - \frac{2}{3}n_f}\,\frac{1}{\ln M^2/\Lambda_{\rm MOM}^2} \qquad \text{(I.45)}$$

where n_f is the number of quark flavors involved, M is the effective momentum scale in the matter and $\Lambda_{\rm MOM}$ is the scale fixing parameter of QCD. The effective momentum scale is estimated to be [Kap79]

$$M^2 = \frac{4}{3}\sum_i \frac{n_i\langle p^2\rangle_i}{\sum_i n_i} \qquad \text{(I.46)}$$

where the sum is over all the constituent species present, each with a number density n_i. $\langle p^2\rangle_i$ is the thermal average of the three momenta of species i. In the case of massless quarks, the above formula reduces to:

$$M^2 = \frac{\frac{4}{3}\Big(16\int_0^\infty dp\,p^4 N_p + 6\sum_f \int_0^\infty dp\,p^4 n_p\Big)}{16\int_0^\infty dp\,p^2 N_p + 6\sum_f\int_0^\infty dp\,p^2 n_p} \qquad \text{(I.47)}$$

where

$$n_p = \frac{1}{\exp\{(p+\mu)/T\}+1} + \frac{1}{\exp\{(p+\mu)/T\}+1}\,, \qquad N_p = \frac{1}{\exp(p/T)-1}\,.$$

The Bose integrals are evaluated using the identity:

$$\int_0^\infty \frac{z^{x-1}}{e^z - 1}\,dz = \Gamma(x)\,\zeta(x)\,x \tag{I.48}$$

where Γ is the factorial function and ζ the Riemann zeta function. The Fermion integrals for arbitrary chemical potential and temperature yield

$$M^2 = \frac{4}{3}\frac{\left(16\cdot 4!\,\zeta(5)\,T^5 + 6\sum_f \int_0^\infty dp\, p^4 n_p\right)}{\left(16\cdot 2\cdot \zeta(3)\,T^3 + 6\sum_f \int_0^\infty dp\, p^2 n_p\right)}. \tag{I.49}$$

They can be solved analytically for the limiting case $T = 0$ and $\mu = 0$ only. For $T = 0$

$$M^2 = \tfrac{4}{5}\,p_F^2\,. \tag{I.50}$$

For $\mu = 0$

$$M^2 = 15.622\,T^2\,. \tag{I.51}$$

For finite μ and T the integrals have to be evaluated numerically. It is interesting to note that the numerical result can be approximated by $M^2 \approx \frac{4}{5}\mu^2 + 15.622\,T^2$. This expression agrees within a few percent with the correct result. The theory therefore has two free parameters, namely the scale fixing parameter Λ_{MOM} and the energy density of the real vacuum, Λ_{VAC}. Λ_{MOM} and Λ_{VAC} can be determined by adjusting the pressure and energy density as calculated in this approach at zero chemical potential to SU(N) Yang Mills Monte Carlo data. One obtains [Cle85] $\Lambda_{MOM} = 100$ MeV and $\Lambda_{VAC} = 190$ MeV/fm^3. These values are often adopted as reference parameters for simple calculations.

The energy density e, entropy density s, and baryon number ρ of the deconfined quark-gluon phase are obtained from the thermodynamical potential via

$$e = -\mu\,\partial\Omega/\partial\mu - T\,\partial\Omega/\partial T + \Omega \tag{I.52}$$

$$S/V = -\,\partial\Omega/\partial T \tag{I.53}$$

$$\rho = -\tfrac{1}{3}\,\partial\Omega/\partial\mu\,. \tag{I.54}$$

The pressure P and energy density e of the plasma tend towards $\pm B_{vac}$, respectively, for $\mu \to 0$. The running coupling constant exhibits, however, a pole at chemical potentials on the order of 100 MeV, so the calculation can not be continued below this value of μ. It is interesting to note that this chemical potential corresponds to zero baryon number density. Hence, the unphysical pole in the coupling constant can be avoided by plotting the thermodynamic variables as a function of the baryon number density ρ.

To do this let us use for simplicity the MIT bag model which describes hadrons and — for this matter — also quark-gluon plasma as a volume in space from which the true vacuum has been expelled — it is filled with color carrying objects, i.e. quarks and gluons, which can not exist in the true vacuum. The bag is a color singlet state, for which a finite energy can be calculated. One can show [Cho74, Chi78] that this simple bag model fits the mass spectrum of the light hadrons quite convincingly, if the following conditions are fulfilled:

(a) The bag, i.e. the volume in which the quarks move, has a constant positive energy density, $B = \Lambda_{\mathrm{VAC}}$. Therefore the total energy increases infinitely with the bag volume. This bag energy accounts for the quark confining potential, which does not allow the separation of single quarks from each other.

(b) The zero point motion has to be included for quarks which move within the small volume of a hadron.

(c) The energy of the quarks is included by solving the Dirac equation for a bound quark state inside the bag.

(d) Low-order terms in the quark-gluon coupling constant are additionally included to take into account the mutual interactions more realistically.

For the extended quark gluon plasma the zero point motion can be neglected, as the bag here is supposed to be much larger than a hadron bag. For the kinetic energy of the quarks the Fermi gas expression for ultra-relativistic particles yields for zero temperature

$$E_{\mathrm{F_Q}} = \tfrac{3}{4}(6\pi^2/g_{\mathrm{Q}})^{1/3}\hbar c \rho_{\mathrm{Q}}^{1/3} . \tag{I.55}$$

From the quark Fermi energy, the Fermi pressure may easily be calculated as

$$P_{\mathrm{F}} = \rho^2 \frac{\partial E}{\partial \rho}\bigg|_{\sigma}$$

which yields

$$P_{\mathrm{FQ}} = \tfrac{1}{4}(6\pi^2/g_{\mathrm{Q}})^{1/3}\hbar c\, \rho_{\mathrm{Q}}^{4/3} . \tag{I.56}$$

Thus, the Fermi energy and -pressure of the quark gas are related via

$$P_{\mathrm{FQ}} = \tfrac{1}{3}\rho_{\mathrm{Q}} E_{\mathrm{Q}} . \tag{I.57}$$

The latter relation hold not only for $T = 0$, but is actually valid for all temperatures, if massless particles are considered. The interaction of the quarks can be calculated from the running coupling and leads to an effective rise of the density-dependent Fermi energy [Stö77].

The density-dependent ground-state energy of the quark bag is then given by

$$E_{\mathrm{BAG}} = \frac{\Lambda_{\mathrm{VAC}}}{\rho_{\mathrm{Q}}} + \frac{3}{4}\left(\frac{6\pi^2}{g_{\mathrm{Q}}}\right)^{1/3} \hbar c(1+\alpha_{\mathrm{c}})\rho_{\mathrm{Q}}^{1/3} = \frac{\Lambda_{\mathrm{VAC}}}{\rho_{\mathrm{Q}}} + E_{\mathrm{F_Q}} . \tag{I.58}$$

E_{BAG} is depicted in fig. I.9. A typical curve for normal nuclear matter is also shown. Observe that near the normal ground state of nuclear matter, with the parameters used by Chodos et al. [Cho74] the quark matter energy is approximately 300 MeV/nucleon above the corresponding nuclear matter curve; here we used $\Lambda_{\mathrm{VAC}} = 56\ \mathrm{V/fm^3}$ and a constant $\alpha_{\mathrm{c}} = 0.5$. However, for smaller B and α_{c} values, this difference is much smaller and the quark energy may be lower than that of nuclear matter at high

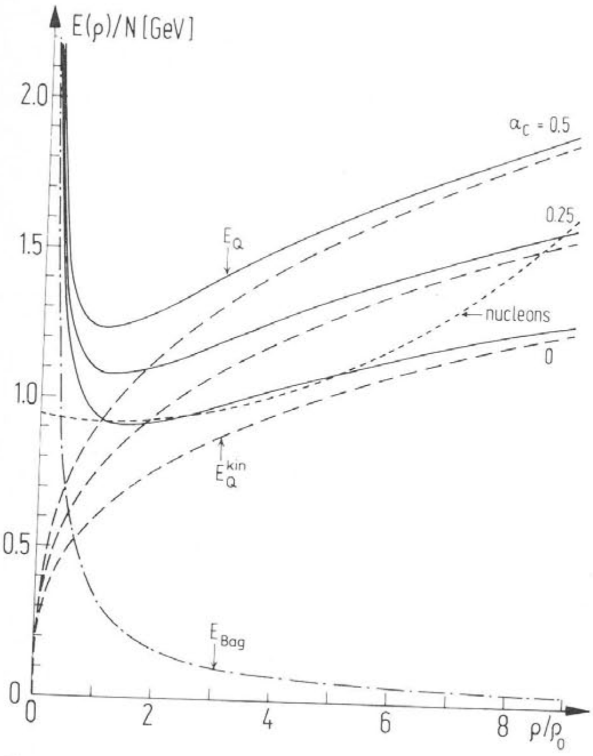

Fig. I.9. Energy per baryon of cold quark matter (solid lines) as compared to nuclear matter (short dashed). The contributions of volume bag energy (dashed-dotted) and kinetic Fermi energy (long dashed) are shown for various strong coupling constants α_c [Stö77b, 80c].

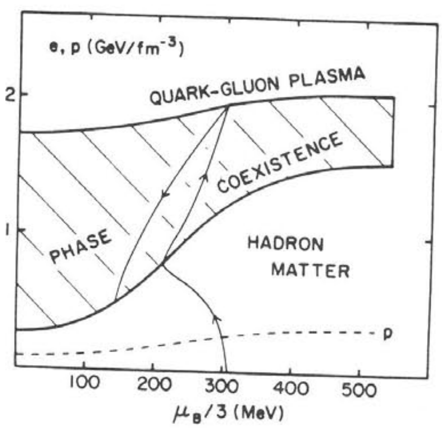

Fig. I.10. Phase coexistence region of the quark-gluon plasma and the hadron plasma in a simple two phase model [Mol84b, Sub85] is shown in the energy density-chemical potential plane. The dynamical path as obtained in the fluid model is sketched by the line with arrowheads.

densities if a rather repulsive interaction is used for the nuclear matter. At moderate densities, however, quark matter is apparently energetically disfavored as compared to ordinary nuclear matter: Other sets of parameters and calculations with running coupling confirm this result [Stö84]; for $\Lambda_{\mathrm{MOM}} = 100$ MeV and $\Lambda_{\mathrm{VAC}} = 190\ \mathrm{MeV/fm^3}$ the minimum energy per baryon of the deconfined phase is about 1.34 GeV, i.e. at an excitation energy per baryon 0.4 GeV higher than the ground state of nuclear matter. Only at high densities would the deconfined state be energetically favorable compared to confined matter at the same density. Absolutely stable quark matter would result, however, if $\alpha = 0$ (see fig. I.9).

The energy density at the crossing of the zero temperature compression energy of the quark matter equation of state with a conventional nuclear compressional energy is (1.4–1.8 GeV/N) · (0.6–1.2 baryons per fm³) = 0.8–2.2 GeV/fm³, hence in the same ball park as the critical energy density obtained from Monte Carlo data at $\mu = 0$. The energy per particle depends on the choice of Λ_{VAC} and Λ_{MOM}. The energy gap is 0.9 GeV/N when Λ_{VAC} is increased to 450 MeV/fm³. These excitation energies may well be achievable in the fragmentation region of ultrarelativistic nuclear collisions and in the central region of stopping collisions.

Figure I.10 shows the finite temperature phase equilibrium calculation done with this simple model [Mol84b, Sub85]. Observe the broad phase coexistence region of the quark-gluon plasma with the hadron plasma. A latent heat of about one to two GeV/fm³ is to be released from the transition from the deconfined phase to the confined phase. This may prove a major handicap for the detection of the quark plasma if it is formed in ultrarelativistic nuclear collisions: Any signal from the interior of the system has to travel through the broad region of the phase coexistence and might be lost by the time it arrives at the surface of the system. This is particularly important for strongly interacting probes like antimatter [Hei84, Sub85] and strangeness [Raf82, Koc83], which will be subject to the complicated hadronic reaction dynamics which has to be tackled on top of the hard problems connected to the confinement problem itself. We will in the next chapter develop the theoretical framework necessary for the quantitative description of the collision dynamics.

II. Many Body Theory of Nuclear Collisions — Finite Time Scales in Finite Systems

II.1. Microscopic kinetic theory

II.1.1. Hierarchy of theories

A comprehensive theory of nuclear collisions at high energies should describe relativistic quantum mechanical wave packets interacting simultaneously with all other wave packets via the correct two nucleon interaction for scattering inside the medium. Although this already neglects correlations, such a quantum mechanical treatment has not yet been attempted, but even the formulation of the interaction itself poses formidable problems. A natural suggestion —and one that has been very successfully employed in the cascade calculations — is to use measured free N–N cross sections as the primary physical input. This is legitimate if only binary N–N interactions occur and the scattered nucleons always reach their asymptotic states before encountering another nucleon; in other words: if the system is dilute. The cascade models and all other models that assume N–N scattering to occur at a point require diluteness.

If one does not want to assume diluteness, the simultaneous interaction of many nucleons has to be allowed. In this case scattering can no longer be described in terms of asymptotic states and cross sections, but an explicit interaction potential is required. The models that use this approach generally describe the nucleon motion in terms of classical trajectories and forces and are therefore often called classical dynamics models. In the relativistic realm there are huge problems even with the formulation of the theory — although it is possible to replace the Dirac equations by relativistic Newton's equations, the meson fields do not obey classical equations even approximately. The only possibility to obtain a solvable model seems to be to ignore second quantization and treat the meson fields classically. The model which comes closest to solving the many-body aspect exactly are the non-relativistic equations of motion with two body potentials, which are actually solvable. The major problem associated with this approach is that classical potentials provide only a poor approximation to N–N scattering and to nuclear binding properties. We will start the discussion of dynamical models with this approach.

II.1.2. Newtonian force model — The classical limit

Consider the classical Γ space description of an A body system with fixed degrees of freedom: we have in mind the colliding system of $A = A_P + A_T$ nucleons. Recall that Γ space is a $6A$ dimensional phase space and the state of the system is represented by one point in this space. Let $\rho(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_A, \boldsymbol{p}_1, \ldots, \boldsymbol{p}_A, t)\, \mathrm{d}\Gamma$ be the probability to find the system at the point $(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_A, \boldsymbol{p}_1, \ldots, \boldsymbol{p}_A)$ in Γ space at time t: ρ is the A-body distribution function. The classical Liouville equation then follows from considering ρ as a probability fluid: