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Abstract

Recent advances in the field of complex, transiently chaotic dynamics are reviewed, based on the results published in the focus issue of *J. Phys. Complex.* on this topic. One group of achievements concerns network dynamics where transient features are intimately related to the degree and stability of synchronization, as well as to the network topology. A plethora of various applications of transient chaos are described, ranging from the collective motion of active particles, through the operation of power grids, cardiac arrhythmias, and magnetohydrodynamical dynamos, to the use of machine learning to predict time evolutions. Nontraditional forms of transient chaos are also explored, such as the temporal change of the chaoticity in the transients (called doubly transient chaos), as well as transients in systems subjected to parameter drift, the paradigm of which is climate change.

1. Introduction

Considering nonlinear dynamical systems, one usually distinguishes between their regular and chaotic behaviours depending on the complexity and predictability of the observed dynamics. In many cases chaotic dynamics appears as an ultimate state of the system, e.g. an attractor with stationary statistical properties remaining unchanged in time. However, chaotic dynamics may also occur in other, transient forms [1], for example, when it exists only for a finite time and disappears eventually (transient chaos in a traditional sense), when it emerges and disappears as a sequence of finite length epochs alternating with a regular dynamics (inter-mittency), or when the properties of a chaotic regime vary due to the drift of system parameter. Being less studied, the latter forms of chaotic dynamics are relevant to a wide variety of phenomena in different fields of science, including chemistry, fluid dynamics, neuroscience, ecology, climate research, and celestial mechanics.

The purpose of this focus issue is to review the most recent achievements in the intriguing field of transient chaos science and stimulate further exploratory studies in the field. Below we provide short summaries of the contributions to the issue, which are grouped in three categories: model-based studies of transient chaos in network dynamics (section 2), examples of transient chaos in various applications (section 3), and new forms of transient chaos (section 4), including doubly transient chaos (section 4.1) and transient chaos appearing in systems subjected to parameter drift (section 4.2).

2. Transient chaos in network dynamics

Network dynamics is one of the simplest mathematical models where transient chaos can be found and studied. Even if such a network consists of identical units, it can exhibit complex transient behaviour, provided the interaction between these units is not too simple and/or the coupling topology of the network has a nontrivial structure.

An instructive example of such type is given in the paper by Clusella and Politi [2], which considers the classical Kuramoto model that describes the dynamics of all-to-all (mean-field) coupled phase oscillators. It turns out that even for identical oscillators, but with certain triharmonic pairwise phase interactions, this

model demonstrates the coexistence of a stable anti-phase two-cluster state with two different types of collective chaotic regimes. One of the regimes is an instance of chaotic itinerancy, while the other has features of chaotic transient. The transient lasts extremely long already in moderate-size systems and seems to persist as a stationary collective chaos in the thermodynamic limit.

The paper by Santos *et al* [3] deals with more complex networks organized as a two-dimensional lattice of adaptive exponential integrate-and-fire neurons. Two types of coupling schemes are considered for this neural network: a regular finite-range nearest-neighbour coupling and a fractal coupling reminiscent of the square Cantor-like ternary set. In both cases, the parameters of the system are identified, for which the system shows rotating spiral-like patterns with randomized core regions—so called spiral wave chimera states. It is expected that these chimera states behave as chaotic transients in networks of small size [4, 5] (although this property is not explored in [3]).

The role of network structure in the generation of complex transients is analyzed in the paper by Ocampo-Espindola *et al* [6]. The paper investigates the dynamical features of the transients from random initial conditions to a fully synchronized (one-cluster) state in different Kuramoto networks. It is shown that for many homogeneous networks (e.g., globally coupled or dense Erdös–Rényi random networks) full synchrony is achieved monotonically, so that the Kuramoto order parameter increases all the time. On the other hand, it is shown that some modular networks with bridge elements demonstrate a non-monotonic variation of the order parameter with local maxima and minima, which indicates a non-monotonic path to synchrony in this case. Theoretical predictions of the paper were also confirmed in experiments with networks of electrochemical reactions.

A qualitatively different problem of network dynamics is addressed in [7]. In this paper, Medeiros *et al* consider a random network consisting of identical discrete-time dynamical units (e.g., Hénon maps). Assuming that the dynamics of each isolated unit is characterized by the coexistence of a chaotic saddle and a periodic attractor in its state-space, the stability of the fully synchronized periodic state is investigated. It is shown that coupling between units can convert the high-dimensional saddle of the entire network into an attractor, where all units are uncorrelated (or in other words desynchronized). This creates the bistability between the synchronized and desynchronized states, which makes the former state extremely vulnerable to any perturbations due to the fractal nature of the saddle's stable manifold.

3. Transient chaos in nature and technology

In this section, we provide an overview of contributions to the focus issue, which describe the emergence of transient chaos in various applications, including self-organization of active particles [8], dynamics of power grids [9], applications in medicine [10] and magnetohydrodynamics [11], as well as in machine learning [12].

The paper by Pikovsky [8] discusses the collective behaviour of chiral active particles, which are discs that move along circles in isolation. The particles interact with elastic repulsive forces, but without any additional alignment force. Starting from random initial conditions, the system relaxes typically to the synchronized state when the velocities of all particles are equal. However, the epoch of normal diffusion, which occurs en route to synchrony, has clear features of supertransient chaos, as the lifetime of chaos grows exponentially with the number of particles. In contrast, with external forcing, traditional transient chaos is observed in the system.

Another application where transient chaos plays an important role is mentioned by Halekotte *et al* in [9]. This paper considers the conditions of stable operation of the power grid described using the Kuramoto model with inertia and the network topology representing the high-voltage transmission grid of the United Kingdom. It is shown that the vulnerability of such power grid with respect to single-node perturbations can be related to the existence and accessibility of chaotic saddles in this dynamical system. The reason is that local basin landscapes of certain nodes show extreme complexity with a very large number of basins, and riddled-like boundaries.

Transient complex spatiotemporal dynamics can also be found in cardiac tissue, where it is associated with cardiac arrhythmias such as ventricular fibrillation. This dysfunction of the heart muscle is studied by Aron *et al* [10] using the 2D Fenton–Karma model, which is a minimal model for the electrophysiology of ventricular myocytes. The paper presents numerical evidence that the average length of the exponentially distributed chaotic transients in the Fenton–Karma model is functionally linked to the features commonly used to characterize excitable media (e.g., the dominant frequency, the size of the excitable gap, or the shape of the action-potential duration restitution curve). The latter can thus be used to predict the average transient time.

The paper by Oliveira *et al* [11] investigates a dynamo model with the helicity as a control parameter. The transition to dynamo occurs via a hysteretic blowout bifurcation associated with a sudden increase in the magnetic energy. In the range between the bifurcation value and the smaller helicity value where boundary crisis takes place, two turbulent attractors coexist (a fully hydrodynamic attractor and a hydromagnetic one) and hysteresis can be observed. Left and right of this window there are chaotic transients, which are governed

by a hydromagnetic and a hydrodynamic chaotic saddle, directing the transients towards the hydrodynamic and the hydromagnetic turbulent attractor, respectively.

There is a recent interest in applying machine learning for predicting chaotic time evolutions. The paper by Kong *et al* [12] shows that via training with measured time series from a small number of distinct bifurcation parameters, taken *solely* from the regime of permanent chaos, reservoir computing machines can generate correct transient chaotic behaviour in the appropriate parameter range. Not only time series but also basic statistical features, such as the lifetime distribution, agree with those of the real system. This machine learning framework can also reproduce intermittency of the target system, and works in the cases where other methods might fail.

4. New forms of transient chaos

In many dynamical systems, as also exemplified above, the occurrence of transient chaos is associated with the existence of chaotic saddles in the corresponding state space. However, this is not the only known mechanism responsible for the emergence of complex transient dynamics. Other possible mechanisms are demonstrated in several papers that deal with the cases of doubly transient chaos and systems subjected to nonrecurrent parameter drift.

4.1. Doubly transient chaos

Chaos in undriven dissipative systems is called doubly transient [13], since the strength of transient chaos changes (typically decreases) over time and all motions eventually stop. Károlyi and Tél show [14] that a clear view of such dynamics is provided by identifying KAM tori or chaotic regions of the dissipation-free case, and following their time evolution in the dissipative dynamics. The tori often smoothly deform first, but later they become disintegrated and dissolve in a kind of shrinking chaos. As a result, the strength of chaos is decreasing, and after a while disappears.

In systems with many degrees of freedom, transient chaos is dominated by normally hyperbolic invariant manifolds (NHIMs) [15]. Motivated by studies of chaotic scattering in Hamiltonian systems, the paper by Jung [16] presents a new version of transient behaviour occurring in a parameter range where NHIMs are already in the process of decay. In such a situation escape takes place from chaotic regions of the original NHIMs which do not even possess a usual stable manifold. Since transient chaos is time-dependent in this range, the phenomenon can also be considered doubly transient, although in a non-dissipative setting.

4.2. Systems subjected to nonrecurrent parameter drift

Systems subjected to parameter drift are of general interest. Cantisán *et al* [17] study the case of a time-delayed oscillator whose time delay varies at a non-negligible rate. They consider a scenario where the time delay is constant at the beginning and end of the observation and increases linearly in the middle. Accordingly, they investigate the transitions from the chaotic attractor that exists for the initial delay value to the attractors that appear for the final delay value. Trajectories from the basin of the initial chaotic attractor are found to tip to periodic states, whose basins are riddled due to the drift, with an accompanying transient chaos. The rate of the parameter drift influences the tipping probability, and the lifetime of transient chaos turns out to decrease with increasing rate.

The problem of parameter drift directs also interest on finite-time properties. Drótos *et al* [18] characterize such properties associated with transient chaos in open dynamical systems: they introduce an escape rate and fractal dimensions suitable for this purpose in a coarse-grained description. The spatial variation of these quantities, especially on long but non-asymptotic integration times, is found to be approximately consistent with the Kantz and Grassberger relation for temporally asymptotic quantifiers. Deviations from this relationship are smaller than differences between various locations.

The paper by Vilela [19] investigates the advection dynamics in incompressible flows with exit regions in the situation when the strength of the flow decays exponentially in time. As a consequence, the fluid energy also decays. The size of the escape regions decreases and eventually is zero, thus the asymptotic flow becomes nonmoving and closed. The fractality of the set occupied by non-escaped fluid tracers is found to increase with time, so that this set asymptotically fills the space. The escape rate is also time-dependent and vanishes asymptotically. Therefore, the described dynamics can be considered as a kind of doubly transient chaos in a low-dimensional non-dissipative system.

We conclude briefly that the contributions to this focus issue show the many-facet nature of transient chaos. They indicate that the exploration of this complex phenomenon is still far from complete and full of surprises [20], which will certainly attract scientific interest for a long time.

Acknowledgments

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