NONLINEAR CRITICAL DYNAMICS OF A SPHERICAL MODEL

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The nonlinear critical slowing down of the order is calculated for the time-dependent Ginzburg–Landau model in the limit of infinite spin dimensionality. The scaling law $\Delta^{(n\ell)} = \Delta^{(\ell)} - \beta$ is verified for this model.

A phenomenological analysis of the nonlinear critical dynamics of purely relaxational systems has led to the scaling law [1,2]

$$\Delta^{(n\ell)} = \Delta^{(\ell)} - \beta\,, \tag{1}$$

relating the critical exponents ($\Delta^{(\ell)}$ and $\Delta^{(n\ell)}$) of the linear and nonlinear relaxation times of the order parameter and the static exponent β. High temperature series [3–6], Monte Carlo works [7] and the only available experiment [8] seem to support this scaling law and its extension [2] to variables other than the order parameter.

One of the assumptions involved in deriving (1) is the scaled form of the relaxation time

$$\tau(\Delta T, M_0) \approx \Delta T^{-\Delta^{(\ell)}} \Phi(M_0/\Delta T^{\beta})\,, \tag{2}$$

where M_0 is the initial deviation of the order parameter M from its final equilibrium value and $\Delta T = |T - T_c|$. Renormalization group calculations [9, 10] have proved this assumption to be correct. The scaling relation (1) however does not follow from (2) unless $\Phi(x) \sim x^{-1}$ for $x \to \infty$. One can give arguments based on ergodicity [1] leading to the x^{-1} behavior, it remains however to be verified by explicit calculation.

In this letter we present the first verification of (1) for a nontrivial model, namely for the time-dependent Ginzburg–Landau (TDGL) model in the limit of infinite spin dimensionality.

In the TDGL model the time evolution of the n components of the order parameter field $S_i = S_i(x, t)$ ($i = 1, ..., n$) is described by the equation [11]:

$$\dot{S}_i = -\Gamma_0 \left(r_0 - \nabla^2 + u \sum_{j=1}^{n} S_j^2 \right) S_i + \eta_i\,, \tag{3}$$

where $\eta_i = \eta_i(x, t)$ is a Gaussian–Markoffian random force [11], $r_0 \sim a + T$ and since we are interested in the $n \to \infty$ limit $u \sim n^{-1}$ is assumed. For simplicity the case $T > T_c$ is considered and we set $\Gamma_0 = 1$.

We shall assume that the heat bath is regulated externally and changes its temperature instantaneously. Then the temperature and consequently the parameters in (3) are well defined. In this case the system may be far from equilibrium in the sense that e.g. M is far from its equilibrium value. This situation is often met in experiments [8, 12].

When calculating the nonlinear relaxation an important point is that the solution of (3) has to be averaged not only over η_i but also over the initial conditions. We shall use a far from equilibrium initial state prepared similarly to the experimental procedure [8, 12]: for $t < 0$ the system is in equilibrium at a temperature $T_1 < T_c$, thus there is a spontaneous order M_0. At $t = 0$ the system in heated instantaneously to $T > T_c$ where the equilibrium value of M is zero.

If the above procedure is used then for $t > 0$ the parameters in (3) have their equilibrium value at $T > T_c$ and the calculation can be carried out as follows. Let the direction $i = 1$ be along the initial ordering. It is known from statics [13] that $M_0 \sim \sqrt{n}$, so it is convenient to write

$$S_1(x, t) = M(t) + L(x, t)\,, \tag{4}$$

where $M(t) = \langle S_1(x, t)\rangle \sim \sqrt{n}$ and the brackets $\langle\,\rangle$ denote the double averaging over the initial conditions and noise as described above. Eliminating $S_1(x, t)$ from (3) the equation $\langle L(x, t)\rangle = 0$ yields in the limit $n \to \infty$:

$$\dot{M}(t) = -\Gamma(t) M(t)\,, \tag{5}$$

$$\Gamma(t) = r_0 + uM^2(t) + un \int_{|q|<\Lambda} \mathrm{d}^d q\, C(q, t)\,. \tag{6}$$

where $C(q, t)$ is the Fourier transform of the transverse correlation function $C(x, t) = \langle S_i(x, t) S_i(0, t)\rangle$ ($i = 2, ..., n$).

The equation for $C(q, t)$ can be obtained by iterating eq. (3) with the $uM^2(t)$ term included in the zeroth order approximation. In the $n \to \infty$ limit one has to keep track of the closed loops since $u \sim n^{-1}$ and each closed loop generates a power of n. The result is the Hartree approximation:

$$C(q, t) = C(q, 0) \exp\left\{-2 \int_0^t [q^2 + \Gamma(s)]\, \mathrm{d}s\right\} + 2 \int_0^t \mathrm{d}t' \exp\left\{-2 \int_{t'}^t [q^2 + \Gamma(s)]\, \mathrm{d}s\right\}. \tag{7}$$

Here the initial correlation is $C(q, 0) = q^{-2}$.

Combining (5), (6) and (7) we find that, as in the case of the ϵ-expansion [10], the equation for $m(t) = M(t)/M_0$ is non-Markoffian

$$-\dot{m} = (r_0 - r_{0c})m + uM_0^2 m^3 + 2unm^3 \int_0^t \mathrm{d}t' \int_{|q|<\Lambda} \mathrm{d}^d q\, q^{-2} \exp[-2q^2(t - t')] \times m^{-3}(t')\dot{m}(t'), \tag{8}$$

where $r_{0c} = r_0(T_c)$ i.e., $r_0 - r_{0c} \sim T - T_c$,

The memory term is crucial for non-classical critical dynamics, if it is neglected the equation of motion becomes that of the mean field theory.

Eq. (8) is a linear integro-differential equation for m^{-2}. It can be solved by Fourier transformation. From the solution it can be easily deduced that near T_c and for small ω the Fourier transform of $m(t)$ has the following scaled form:

$$\tilde{m}(\omega) = \int_0^\infty \mathrm{e}^{\mathrm{i}\omega t} m(t)\, \mathrm{d}t = \frac{1}{r} f\left(\frac{\omega}{r}, \frac{M_0}{\Delta T}\right), \tag{9}$$

where r is the inverse susceptibility $r \sim \chi^{-1} \sim \Delta T^{2/(d-2)}$. From (9) the scaled form (2) of the relaxation time $\tau = \tilde{m}(0)$ follows and one can read the well known [11] value of the critical exponent of the linear relaxation time $\Delta^{(\ell)} = 2/(d-2)$.

The exponent of the nonlinear relaxation time is found by calculating $\tilde{m}(\omega)$ at T_c with $M_0 \neq 0$. One can show that for $2 < d < 4$, $\tilde{m}(\omega) \sim \omega^{(d-6)/4}$. Then scaling (9) implies $\tau \sim \Delta T^{(d-6)/2(d-2)}$ and so $\Delta^{(\mathrm{n}\ell)} = (6-d)/2(d-2)$. Since $\Delta^{(\ell)} = 2/(d-2)$ and $\beta = \frac{1}{2}$ we have proved the validity of the scaling law (1) in the $n \to \infty$ limit of the TDGL model. Finally we note that for $d > 4$ the memory effects are negligible and the molecular field results [1] apply.

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