Amplified internal pulsations on a stratified exchange flow excited by interaction between a thin sill and external seiche

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Exchange flow over a thin sill is studied in a laboratory tank. Characteristic pulsation in the flow intensity is visualized and measured in the vicinity of the sill. The frequency of pulsation is identified as the first eigenmode of external seiche (sloshing) of the whole water body. Control experiments in a simplified setup unambiguously confirmed that initial configurations with horizontal pressure gradient always generate such oscillations. The almost invisible, small-amplitude vertical vibration is magnified by the sill generating internal waves in the pycnocline. These results can serve to increase awareness, also among researchers analyzing field data, of the potential for external seiching to drive internal flow characteristics. © *2007 American Institute of Physics*. [DOI: 10.1063/1.2796182]

Internal dynamical processes in semi-enclosed natural water bodies such as bays, gulfs, or harbors have significant impacts on stratification and mixing at large depths (see, e.g., Ref. 1). A horizontal density gradient can drive circulation within the enclosure and maintain an exchange of fluid with adjoining reservoirs.[2] Even in the case of simple basin geometries, e.g., bottom topography, an isolated sill enhances the complexity of flow phenomena.[3] The most essential aspects of exchange flows have been successfully explained by relatively simple two-dimensional models, e.g., for the Knight Inlet[4] or the Strait of Gibraltar.[5] The formulation of theoretical and laboratory models is further simplified when a few homogeneous fluid layers of sharp density interfaces are considered.[3]

Here we report on a particular experimental result in a laboratory model of three-layer exchange flow over a sill. A characteristic pulsation in the intensity of the exchange flow is found in the vicinity of the sill. We attribute this effect to the influence of the external seiche (see below) affecting the internal dynamics.

The setup is depicted in Fig. 1(a). The total length of the tank is 11 m, which can be sectioned by Plexiglas lock gates into nine compartments in several combinations. The width and height of the tank are 15 and 25 cm, respectively, and the walls are made of glass. In the middle of the tank, a split lock gate was installed, where the fixed bottom part formed a thin sill (of thickness 5 mm), and the top part was removable. The height of the sill was $h=7.5$, 9.0, or 12.0 cm. The total length of the two basins was $L=L_l+L_r=3,4,\ldots,7$ m, where L_l and L_r denote the length of the left and right sections [Fig. 1(a)]. The left compartment was filled up to the sill height $(H_1 \equiv h)$ by salt solution of density $\varrho_1 = 1023.0 \pm 0.7$ g/cm^3. A more diluted liquid ($\varrho_2 = 1013.0 \pm 0.8$ g/cm^3) was carefully layered on top up to a height $H_2=5.0-12.0$ cm. The right basin was filled up by tap water ($\varrho_0 = 998.2 \pm 0.7$ g/cm^3) precisely up to the height of $H \equiv H_1+H_2$ in order to avoid surface disturbances. Dye was also added to all solutions, so as to obtain decreasing darkness with decreasing density (see Fig. 1). The initial solutions were kept at rest for a couple of hours in order to minimize temperature effects. The upper part of the lock gate was pulled up to initiate the exchange flow.

Note that this setup produces flows of zero net flux, that is, the total volume of fluid and the surface height are constant in both basins. The flow structure shortly after the start is depicted in Fig. 1(b), and is photographed at the vicinity of the sill in Fig. 1(c). The lightest fluid of density ϱ_0 forms a "standard" gravity current at the top. The salt solution of density ϱ_2 immediately begins to move in the opposite direction, it tumbles over the sill, and descends to the bottom of the right basin. Moreover, the strong shear continuously drags a finite volume of the densest fluid (ϱ_1) across the sill, thus a high-density gravity current on the right side is formed by the mixture of both salt solutions and the fresh water [appearing in green in Fig. 1(c)]. This overall pattern of flow in each of the layers remains more or less steady until the light gravity current approaches the sill again after bouncing back from the end wall of the channel. Several well-known phenomena are clearly observable during the experiments. Besides the gravity currents, supercritical flows, internal hydraulic jumps, entrainment, strong turbulent mixing, internal waves and solitons propagating in both directions along the

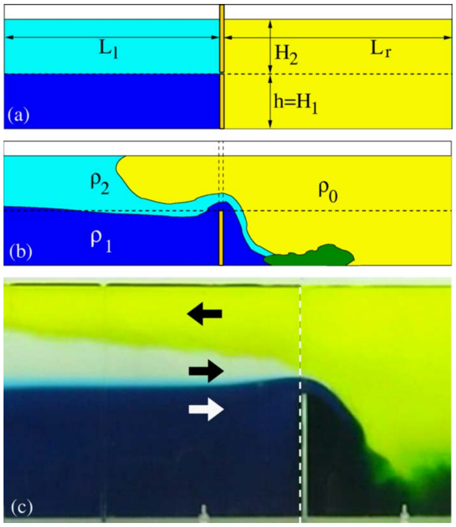

FIG. 1. (Color online) (a) Initial setup of a zero flux exchange flow experiment. Geometric parameters: width of left (right) compartments L_l (L_r), sill height h, and depth of layers H_1 and H_2. (b) Sketch of the flow structure after opening the lock gate ($\varrho_1 > \varrho_2 > \varrho_0$, see text). (c) The flow pattern recorded in the vicinity of the sill at 7.2 s after opening the lock gate. Arrows indicate the flow directions. The vertical dashed line marks the position of layer depth measurements for the densest bottom fluid of ϱ_1 (enhanced online).

density interfaces, internal bores (after reflections of gravity currents at the end walls), and internal seiche (see below) in the left basin all appear before the final equilibrium sets in. The richness of flow features and the large parameter space ($\varrho_1, \varrho_2, h, H_1, H_2, L_l, L_r$) require a long systematic study to extract quantitative results. Here we concentrate on a single aspect that, to the best of our knowledge, has not been considered in the literature in the context of pure exchange flows.

Figure 1(c) shows a snapshot made by a video camera at the sill region. The video records were sliced (25 frames per second), converted to grayscale images, and evaluated quantitatively by simple procedures. The height of the dense layer at a given horizontal position was acquired by detecting the sharp change in the grayness level at the internal boundary. The results exhibit marked oscillations revealed by a periodic change of layer depths. Figure 2 illustrates that the oscillation frequency does not depend on the height of the sill h at constant total fluid depth $H=H_1+H_2$, but the amplitude and the average layer depth do. Further tests showed that the key parameter determining the frequency is the total length of the tank $L=L_l+L_r$, see Fig. 3. Note that the equality of basin lengths $L_l=L_r$ is not required.

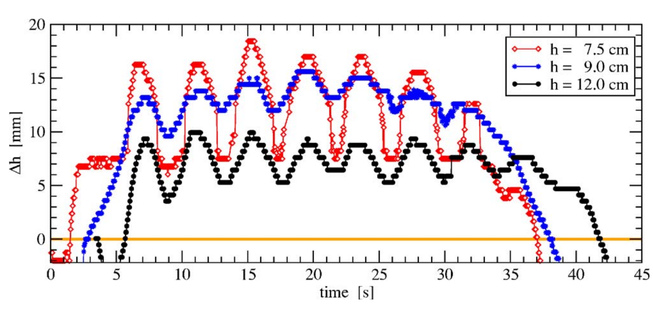

FIG. 2. (Color online) Oscillation of the densest layer depth as a function of time at the position shown in Fig. 1(c). The records for the $h=9$ and 12 cm experiments (blue stars and black circles) are shifted in time to illustrate the match of frequencies. The horizontal line indicates the height of the sill. The total fluid depth and the length of the tank were $H=19.0$ cm and $L=276.0$ cm in each experiment.

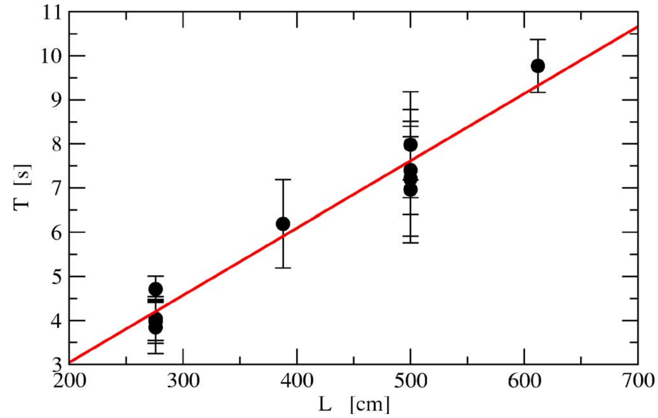

FIG. 3. (Color online) Oscillation period T of the bottom layer depth as a function of the total length $L=L_l+L_r$ for 12 exchange flow experiments of various total fluid depths $H=14.0-21.0$ cm. The slope of the line is 0.0152 s/cm.

The linear length dependence of the oscillation period (Fig. 3) suggested that the flow pulsation is related to the well-known standing wave phenomenon of fluid bodies in closed or partially closed basins, called "seiche" (sloshing). The term was coined by Forel, who had observed the effect in the late 1880s, in Lake Geneva, Switzerland.[6] When some external disturbance moves the water body out of gravitational equilibrium, the natural response is an oscillatory motion with period

$$T = \frac{2L}{n\sqrt{gH}}, \tag{1}$$

where L is the size of the (closed) basin, H is the water depth, g is the gravitational acceleration, and $n=1,2,\ldots$ is the number of nodes in a given mode of standing waves. Natural waters are typically density stratified,[7] therefore internal seiches (standing waves of isopycnals) are also frequently present. Such oscillations are usually generated by wind forcing,[8–10] atmospheric pressure variations,[11] seismic activity,[12] tidal effects,[13,14] or even tsunamis,[15] and they are routinely observed in a wide variety of natural water bodies.

In order to confirm that the detected pulsations are determined by the fundamental mode of external seiche, we performed control experiments in a simplified setup. Here the exchange flow was switched off by filling the tank with two fluids, a dense bottom layer of density ϱ_1 up to the

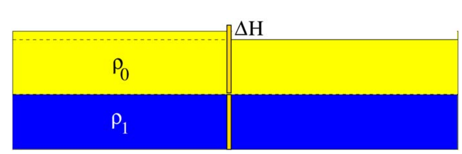

FIG. 4. (Color online) Initial setup of the control experiments.

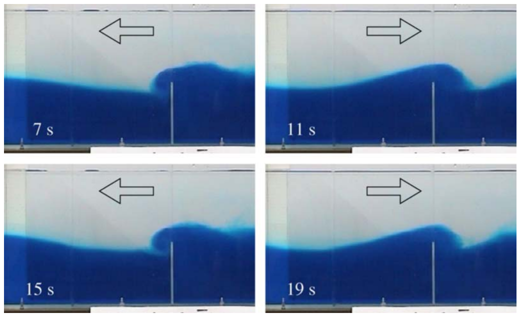

FIG. 5. (Color online) Internal waves generated by the sill in the control experiment after pulling up the lock gate. Time and flow direction are indicated. $L=500$ cm, $h=9.0$ cm, $H=17.0$ cm, and $\Delta H=2$ mm. Note the lack of visible changes on the top surface.

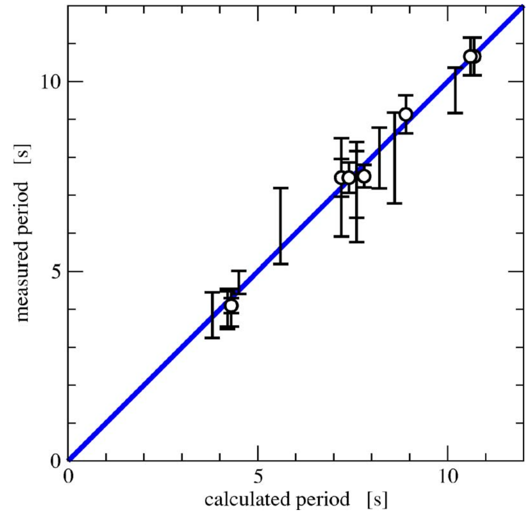

FIG. 7. (Color online) Correlation plot of the theoretical [see Eq. (1)] and measured periods for each experiment. White symbols denote the control results with the two-layer setup, the rest is for the three-layer experiments. Error bars are estimated from the half-width of the main Fourier peak shown in Fig. 6(b).

height of the sill h, and tap water on top of it. The only difference between the two basins was a water level slightly higher by $\Delta H=1,2$, or 3 mm at one side (see Fig. 4).

Figure 5 illustrates the flow patterns after opening the lock gate. The initial tiny difference of water levels excited an almost invisible external seiche. However, a small vertical oscillation can produce relatively strong horizontal motion, especially in a shallow fluid near a node of a standing wave. The horizontal flow is blocked by the thin sill in the bottom layer, therefore a strong shear arises similarly to the three-layer case of Fig. 1(c). The shear flow drags the heavy fluid and forces it to tumble over the sill periodically. This tumbling fluid generates marked internal waves along the pycnocline that move in the direction of the flow. We emphasize that a careful leveling in both basins ($\Delta H=0$) inhibits external seiche, and thus internal wave generation over the sill.

The time evolution of the dense layer depth was evaluated by the same method as before. A typical time series is shown in Fig. 6(a). The oscillation is not entirely clear, because the internal waves are reflected from the end walls, and the superposition of such waves results in a modulation of the main signal. The power spectrum [Fig. 6(b)] helps to identify the basic frequencies: the largest peak at $T_1=7.4$ s belongs to the fundamental mode of external seiche [$n=1$ in Eq. (1)], and the second mode with $n=2$ ($T_2=3.7$ s) is clearly detectable. We repeated the control experiment with different total depths H, different total lengths L (of either symmetric or asymmetric L_l and L_r combinations), and different (but small) initial excess heights ΔH.

The summary of the results for both sets of experiments is shown in Fig. 7, where the simple formula Eq. (1) is tested against the measured periods. We think that the conclusion is convincing; the pulsation in the three-layer flow experiments is also a consequence of seiche oscillations excited by the initial hydrostatic pressure difference between the two basins.

We note that the thickness of the sill is not a key parameter to control the pulsations. Experiments with two wide obstacles (rectangular cross section of 12.0×7.5 cm^2 and triangular cross section of height 9.0 cm and baseline 14.0 cm) resulted in different flow details, but the frequency of external seiche could be unambiguously extracted over the sill.

The first hint to internal waves generated by external seiche over topography was published by Zeilon in 1913; see Refs. 16 and 17. Later, model studies by New and Dyer[18] suggested that field observations[19] in Southampton Water (north of the Isle of Wight) could be explained by such an effect. Münnich[20] considered continuous stratification and a sill for a two-basin closed reservoir. He found that the oscillation mode spectrum of a lake with continuous stratification is dense for a rectangular basin, which implies that such a

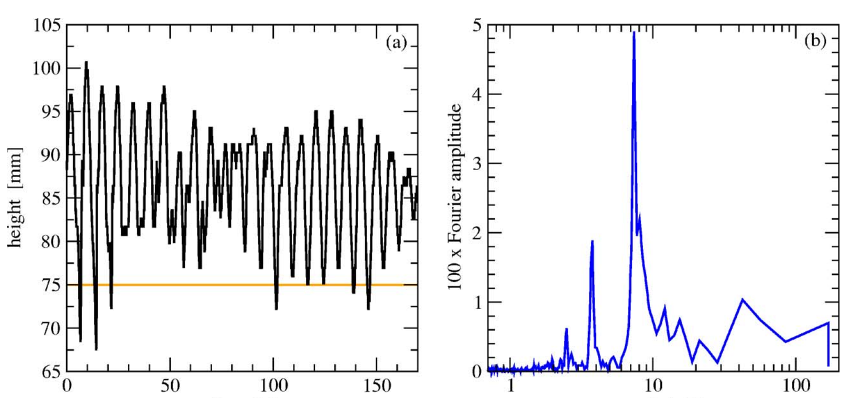

FIG. 6. (Color online) (a) Height of the dense bottom layer as a function of time measured 8 mm away from the sill. Horizontal line indicates the sill height. $L=612$ cm, $h=7.5$ cm, $H=18.0$ cm, and $\Delta H=2$ mm. (b) Power spectrum of the signal shown in (a) as a function of period. (Note the logarithmic horizontal scale.)

lake can oscillate essentially at any frequency. However, higher modes can be excited only with a complicated external mechanism (far distinct from a homogeneous wind stress), and such modes are damped faster because of the higher related shear stresses.[20] Continuous stratification is not a prerequisite for complicated oscillations; even a few homogeneous layers can exhibit standing waves of various modes.[21,22] Internal waves over a sill and damping of external seiches are exhaustively discussed by Parsmar and Stigebrandt[16] and Stigebrandt.[17] They pointed out that the most efficient mechanism for energy dissipation of barotropic oscillations is baroclinic wave generation, especially in fjords where the nodal line (with maximum horizontal transport) is at the mouth. This damping mechanism is less effective in inland lakes, where the node of the fundamental seiching mode is in the middle, where often the cross-sectional area is maximal and the bottom topography is smooth.[16]

Frequency domain analysis is widely used to evaluate internal dynamics in stratified fluids. We think that external seiche is not always considered as a source of internal waves. One example is represented by Fig. 11 in the work by Zhu *et al.*,[23] where exchange flow experiments over various openings are reported. The basic frequency f_1 is identified with an internal seiche, and neighboring Fourier peaks with the higher harmonics of f_1. An additional peak at 0.35 Hz is not evaluated, although this frequency agrees completely with the fundamental mode of external seiche in their setup.

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