1.

Általános útmutatás: olyan sorrendben számoljuk ki a „valami(t)" függvényeket, amilyenben ez lehetséges.

$$v_y = v_{fy} + v_{rel\,y} = \frac{1}{2} b_0 t^2$$

$$y(t) = y(t_0) + \int_{t_0}^{t} v_y(t')\,dt' = -l + \int_0^t \frac{1}{2} b_0 t'^2\,dt' =$$

$$= -l + \frac{1}{2} b_0 \left[\frac{t'^3}{3}\right]_0^t = -l + \frac{1}{6} b_0 t^3 \qquad (*)$$

$$v_x = v_{fx} + v_{rel\,x} = v_0 \left(\frac{y}{l} + 1\right)\left(1 - \frac{y^2}{l^2}\right) + a_0 t$$

$$x(t) = x(t_0) + \int_{t_0}^{t} v_x(t')\,dt' = \int_0^t \left(v_0 \left(\frac{y(t')}{l} + 1\right)\left(1 - \frac{y(t')^2}{l^2}\right) + a_0 t' \right) dt' =$$

$$\underset{(*)}{=} \int_0^t \left(v_0 \left(\frac{-l + \frac{1}{6} b_0 t'^3}{l} + 1\right)\left(1 - \frac{\left(-l + \frac{1}{6} b_0 t'^3\right)^2}{l^2}\right) + a_0 t' \right) dt' =$$

$$= \int_0^t v_0 \frac{1}{6} \frac{b_0}{l} t'^3 \left(1 - 1 + \frac{1}{3} \frac{b_0}{l} t'^3 - \frac{1}{36} \frac{b_0^2}{l^2} t'^6\right) dt' + \int_0^t a_0 t'\,dt' =$$

$$= \int_0^t \frac{1}{6} v_0 \frac{b_0}{l} \cdot \frac{1}{3} \frac{b_0}{l} t'^6\,dt' - \int_0^t \frac{1}{6} v_0 b_0 \cdot \frac{1}{36} \frac{b_0^2}{l^2} t'^9\,dt' + \int_0^t a_0 t'\,dt' =$$

$$= \frac{1}{3 \cdot 6 \cdot 7} \frac{v_0 b_0^2}{l^2} t^7 - \frac{1}{6 \cdot 36 \cdot 10} \frac{v_0 b_0^3}{l^3} t^{10} + \frac{1}{2} a_0 t^2$$

$$⊛ \rightarrow \quad t = \sqrt[3]{6\,\frac{y+l}{b_0}}$$

$$\rightarrow \quad x(y) \equiv x(t(y)) = \frac{1}{3\cdot6\cdot7}\,\frac{v_0 b_0^2}{l^2}\left(6\,\frac{y+l}{b_0}\right)^{\frac73} - \frac{1}{10\cdot6\cdot36}\,\frac{v_0 b_0^3}{l^3}\left(6\,\frac{y+l}{b_0}\right)^{\frac{10}{3}} +$$

$$+ \frac12 a_0 \left(6\,\frac{y+l}{b_0}\right)^{\frac23}$$

$$X \equiv x(y=l) = \frac{1}{3\cdot6\cdot7}\,\frac{v_0 b_0^2}{l^2}\left(6\,\frac{2l}{b_0}\right)^{\frac73} - \frac{1}{6\cdot360}\,\frac{v_0 b_0^3}{l^3}\left(6\,\frac{2l}{b_0}\right)^{\frac{10}{3}} +$$

$$+ \frac12 a_0\left(6\,\frac{2l}{b_0}\right)^{\frac23} = \frac{2^{\frac73}\cdot6^{\frac43}}{3\cdot7}\,\frac{v_0\, l^{\frac13}}{b_0^{\frac13}} - \frac{2^{\frac{10}{3}}\cdot6^{\frac13}}{10}\,\frac{v_0\, l^{\frac13}}{b_0^{\frac13}} +$$

$$+ \frac{6^{\frac23}}{2^{\frac13}}\, a_0\, \frac{l^{\frac23}}{b_0^{\frac23}} = \left(\frac{2^{\frac73}\cdot6^{\frac43}}{3\cdot7} - \frac{2^{\frac{10}{3}}\cdot6^{\frac13}}{10}\right)\frac{v_0\, l^{1/3}}{b_0^{1/3}} + \frac{6^{\frac23}}{2^{\frac13}}\,\frac{a_0\, l^{2/3}}{b_0^{2/3}}$$

2. $t_0=0$, $r(t=0)=r_0$, $\varphi(t=0)=0$

$$v_{f\varphi}=0 \Rightarrow \omega_f \equiv \dot{\varphi}_f=\frac{v_{f\varphi}}{r}=0 \quad \text{minden } r \text{ mellett}$$

$$\dot{\varphi}=\dot{\varphi}_f+\dot{\varphi}_{rel}=0+\omega_0=\omega_0$$

$$\varphi(t)=\varphi(t_0)+\int_{t_0}^{t}\dot{\varphi}(t')\,dt'=0+\int_0^t \omega_0\,dt'=\omega_0 t \quad (*)$$

$$v_r \underset{\uparrow}{=} v_{fr}=v_0\left(1-\frac{(\varphi-\pi/8)^2}{(\pi/8)^2}\right)$$

a feladat szövege szerint

$$r(t)=r(t_0)+\int_{t_0}^{t}v_r(t')\,dt'=r_0+\int_0^t v_0\left(1-\frac{(\varphi(t')-\pi/8)^2}{(\pi/8)^2}\right)dt'=$$

$$\underset{\substack{\uparrow\\(*)}}{=} r_0+\int_0^t v_0\left(1-\frac{(\omega_0 t'-\pi/8)^2}{(\pi/8)^2}\right)dt'=r_0+$$

$$+v_0\int_0^t\left(-\left(\frac{8}{\pi}\omega_0\right)^2 t'^2+\frac{16}{\pi}\omega_0 t'\right)dt'=r_0+v_0\left(-\frac{64}{3\pi^2}\omega_0^2 t^3+\right.$$

$$\left.+\frac{8}{\pi}\omega_0 t^2\right)$$

$$(*) \rightarrow t=\frac{\varphi}{\omega_0}$$

$$\rightarrow r(\varphi)\equiv r(t(\varphi))=r_0+v_0\left(\frac{8}{\pi}\cancel{\omega_0}\frac{\varphi^2}{\omega_0^{\cancel{2}}}-\frac{64}{3\pi^2}\cancel{\omega_0^2}\frac{\varphi^3}{\omega_0^{\cancel{3}}}\right)$$

$$R=r\left(\varphi=\frac{\pi}{4}\right)=r_0+\frac{v_0}{\omega_0}\left(\frac{8}{\pi}\cdot\frac{\pi^2}{4^2}-\frac{64}{3\pi^2}\cdot\frac{\pi^3}{4^3}\right)=$$

$$=r_0+\frac{v_0}{\omega_0}\left(\frac{\pi}{2}-\frac{\pi}{3}\right)=r_0+\frac{\pi}{6}\frac{v_0}{\omega_0}$$

3.

$$v_x = v_{fx} + v_{rel\,x} = \cancel{v_0} + \frac{1}{2} b_0 t^2 - \cancel{v_0} = \frac{1}{2} b_0 t^2$$

$$x(t) = x(t_0) + \int_{t_0}^{t} v_x(t')\,dt' = \int_0^t \frac{1}{2} b_0 t'^2\,dt' = \frac{1}{6} b_0 t^3$$

$$a_y = a_{fy} + a_{rel\,y} = 0 + a_0 \frac{x^2}{\ell^2} = a_0 \frac{x^2}{\ell^2}$$

$$v_y(t) = v_y(t_0) + \int_{t_0}^{t} a_y(t')\,dt' = \int_0^t a_0 \frac{x(t')^2}{\ell^2}\,dt' =$$

$$= \int_0^t a_0 \frac{1}{6^2} b_0^2 t'^6 \frac{1}{\ell^2}\,dt' = \frac{1}{7\cdot 6^2} \frac{a_0 b_0^2}{\ell^2} t^7$$

$$y(t) = y(t_0) + \int_{t_0}^{t} v_y(t')\,dt' = -\ell + \int_0^t \frac{1}{7\cdot 6^2} \frac{a_0 b_0^2}{\ell^2} t'^7\,dt' =$$

$$= -\ell + \frac{1}{8\cdot 7\cdot 6^2} \frac{a_0 b_0^2}{\ell^2} t^8$$

$$\rightarrow \quad t(y) = \sqrt[8]{8\cdot 7\cdot 6^2 \frac{(y+\ell)\ell^2}{a_0 b_0^2}} \quad , \quad t(y=\ell) = \sqrt[8]{2\cdot 8\cdot 7\cdot 6^2 \frac{\ell^3}{a_0 b_0^2}}$$

$$X = x(t(y=\ell)) = \frac{1}{6} b_0 \left(2\cdot 8\cdot 7\cdot 6^2 \frac{\ell^3}{a_0 b_0^2}\right)^{3/8} =$$

$$= \frac{1}{6} b_0 \left(2^6\cdot 3^2\cdot 7 \frac{\ell^3}{a_0 b_0^2}\right)^{3/8}$$

4.

$$\dot{\varphi}(t) = \frac{v_\varphi(t)}{r(t)} = \frac{v_{f\varphi}(t) + v_{rel\,\varphi}(t)}{r(t)} =$$

$$= \frac{0 + u_1 \frac{r(t)}{R_1}}{r(t)} = \frac{u_1}{R_1}$$

$$\varphi(t) = \varphi(t_0) + \int_{t_0}^{t} \frac{v_\varphi(t')}{r(t')}\,dt' = \varphi(t_0) + \int_{t_0}^{t} \frac{u_1}{R_1}\,dt' =$$

$$= 0 + \int_0^t \frac{u_1}{R_1}\,dt' = \frac{u_1 t}{R_1}$$

$$v_r = v_{fr} + v_{rel\,r} = v_0 \sin^2\varphi + a_0 t$$

$$r(t) = r(t_0) + \int_{t_0}^{t} v_r(t')\,dt' = R_1 + \int_0^t \left(v_0 \sin^2\varphi(t') + a_0 t'\right)dt' =$$

$$= R_1 + \int_0^t \left(v_0 \sin^2\left(\frac{u_1 t'}{R_1}\right) + a_0 t'\right)dt' =$$

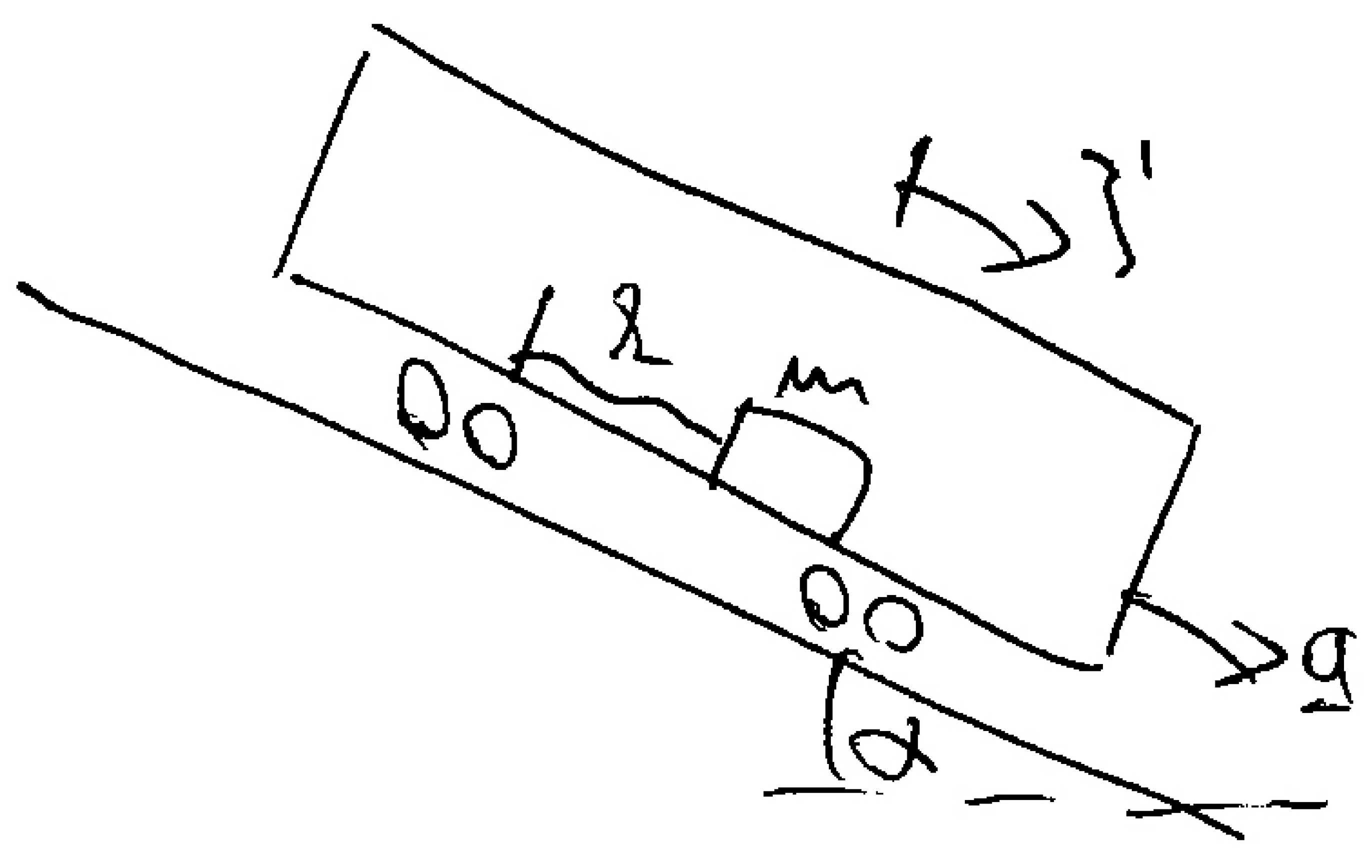
$$= R_1 + \int_0^t \left(\frac{v_0}{2} - \frac{v_0}{2}\cos\left(\frac{2u_1 t'}{R_1}\right) + a_0 t'\right)dt' =$$

$$= R_1 + \left[\frac{v_0 t'}{2} - \frac{v_0}{2}\cdot\frac{R_1}{2u_1}\sin\left(\frac{2u_1 t'}{R_1}\right) + \frac{1}{2}a_0 t'^2\right]_0^t =$$

$$= R_1 + \frac{v_0 t}{2} + \frac{1}{2}a_0 t^2 - \frac{v_0 R_1}{4u_1}\sin\left(\frac{2u_1 t}{R_1}\right)$$

EMB/Gy1/06

5.

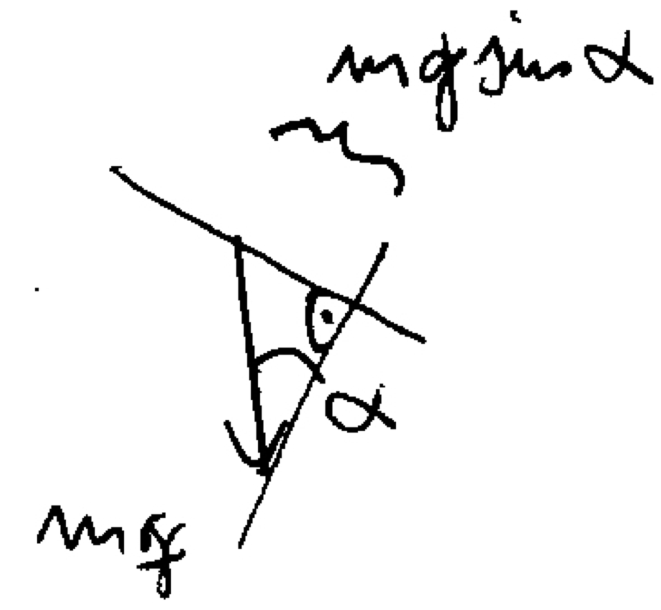


a) $(m\ddot{\underline{r}}')_{||} = (\underline{F}_{rug})_{||} + (m\underline{g})_{||} - (m\underline{a}_0)_{||}$

$(m\ddot{\underline{r}}')_{||} = m\ddot{\xi}'$

$(\underline{F}_{rug})_{||} = -k\xi'$

$(m\underline{g})_{||} = mg\sin\alpha$



$(-m\underline{a}_0)_{||} = -ma_0$

→

$m\ddot{\xi}' = -k\xi' + mg\sin\alpha - ma_0$

b)

$\ddot{\xi}' \overset{!}{=} 0$

→

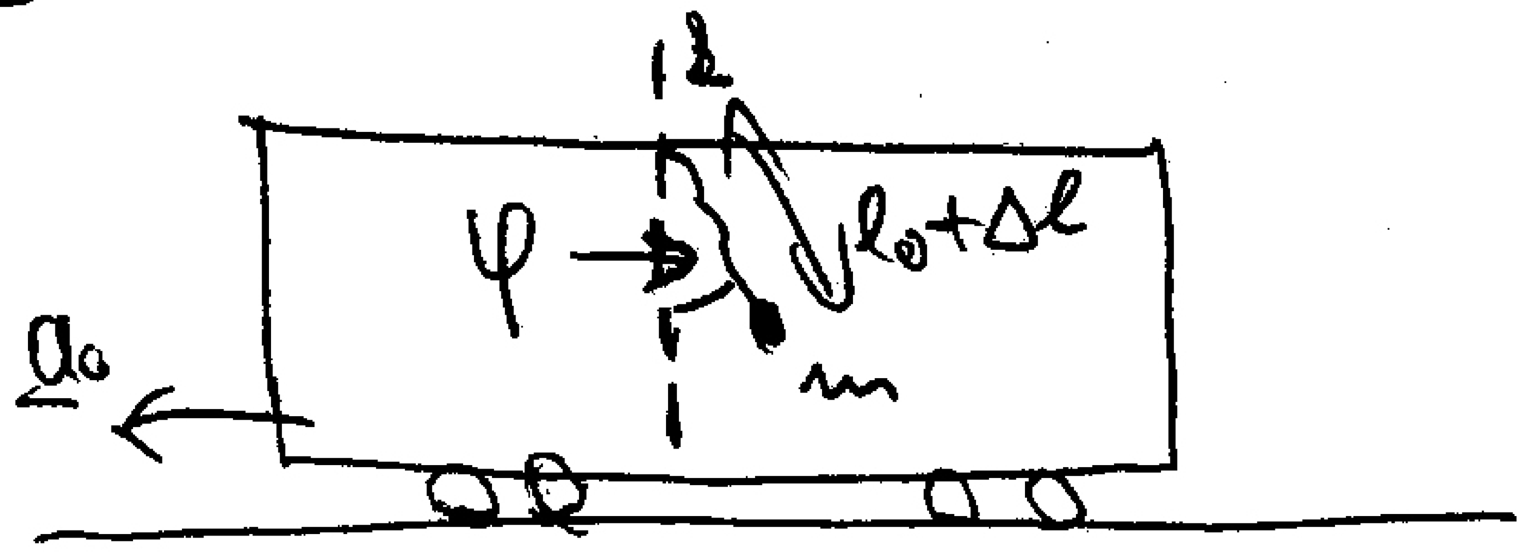
$0 = -k\xi'^{*} + mg\sin\alpha - ma_0$

$\xi'^{*} = \frac{m(g\sin\alpha - a_0)}{k}$

6.

EMB/Gy1/07

$\varphi^* = ?$ $\Delta l^* = ?$

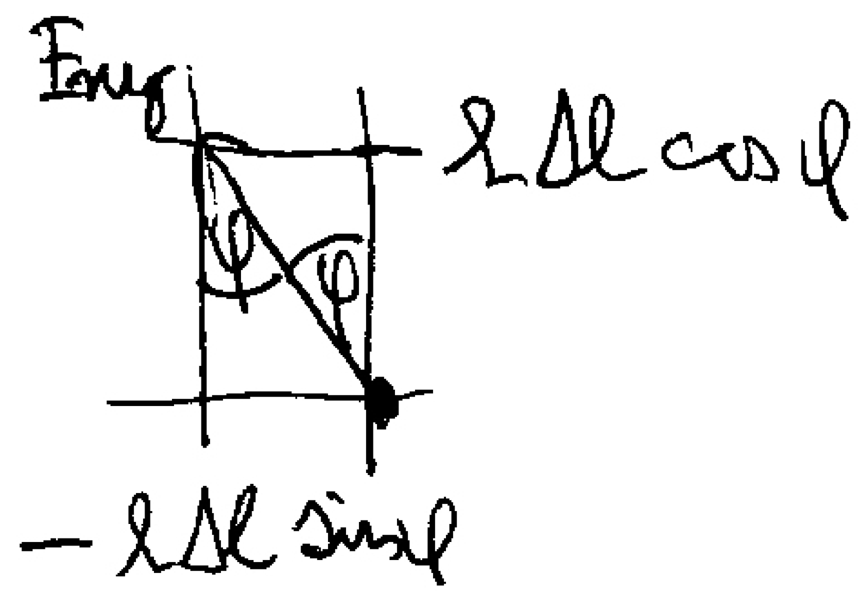


$$m\ddot{\underline{r}}' = \underline{F}_{rug} + m\underline{g} - m\underline{a}_0$$

$$(m\ddot{\underline{r}}')_{x'} = m\ddot{x}'$$
$$(m\ddot{\underline{r}}')_{y'} = m\ddot{y}'$$

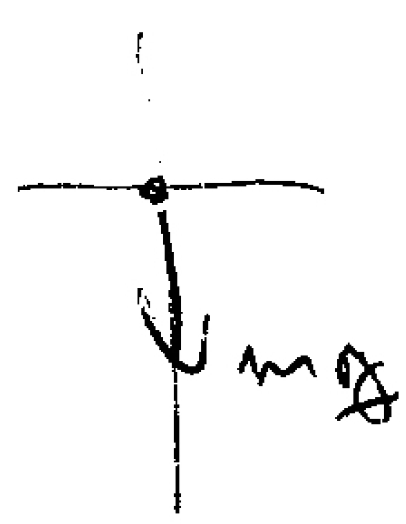
$$(\underline{F}_{rug})_{x'} = -s\Delta l\sin\varphi$$
$$(\underline{F}_{rug})_{y'} = +s\Delta l\cos\varphi$$

ha $\Delta l > 0$:

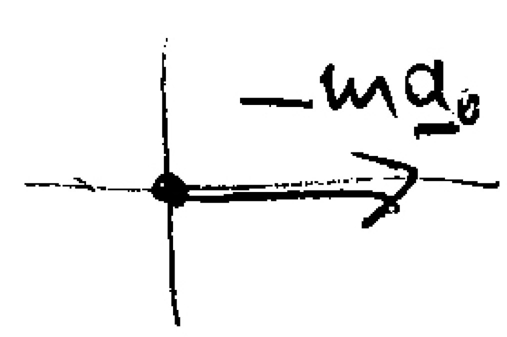


$\Delta l < 0$-ra hasonló

$$(m\underline{g})_{x'} = 0$$
$$(m\underline{g})_{y'} = -mg$$



$$(-m\underline{a}_0)_{x'} = ma_0$$
$$(-m\underline{a}_0)_{y'} = 0$$



$$\Rightarrow \quad m\ddot{x}' = -s\Delta l\sin\varphi + ma_0$$
$$m\ddot{y}' = s\Delta l\cos\varphi - mg$$

$$s^2\Delta l^{*2}(\underbrace{\sin^2\varphi^* + \cos^2\varphi^*}_{=1}) = m^2a_0^2 + m^2g^2$$
$$\Delta l^* = \frac{m}{s}\sqrt{a_0^2+g^2}$$

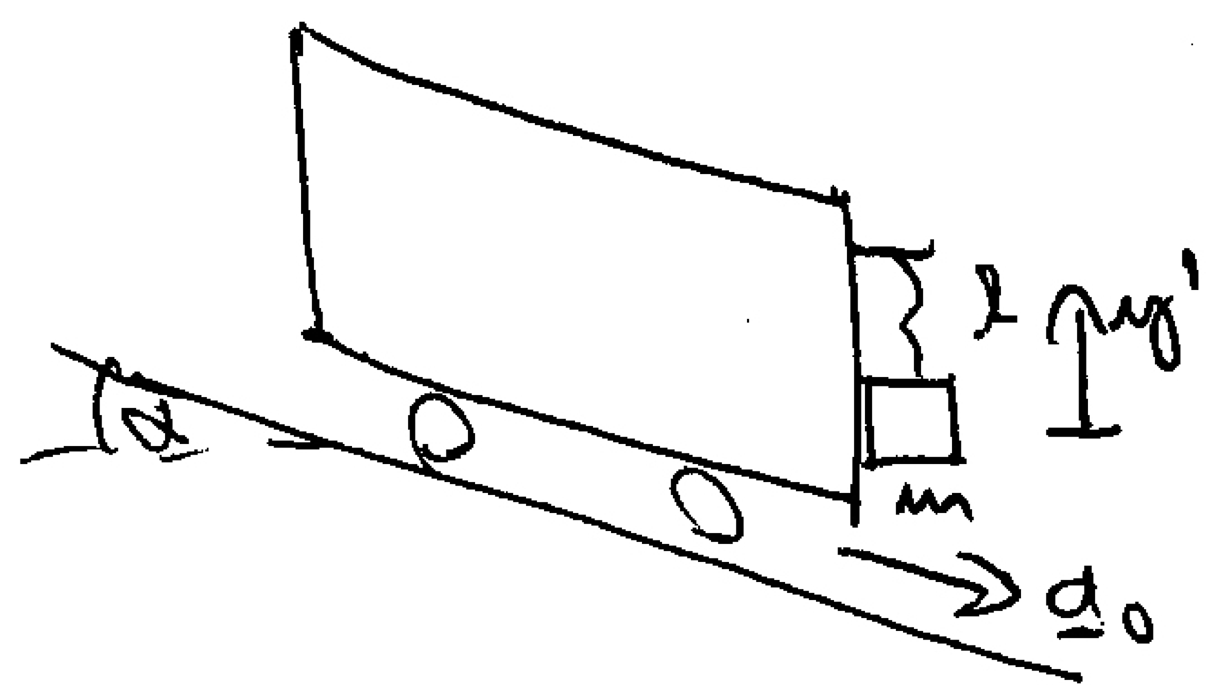
$\ddot{x}' \overset{!}{=} 0,\ \ddot{y}' = 0$

$$\Rightarrow \quad 0 = -s\Delta l^*\sin\varphi^* + ma_0$$
$$0 = s\Delta l^*\cos\varphi^* - mg$$

$$\frac{s\Delta l^*\sin\varphi^*}{s\Delta l^*\cos\varphi^*} = \frac{ma_0}{mg}, \quad \mathrm{tg}\,\varphi^* = \frac{a_0}{g}, \quad \varphi^* = \mathrm{arctg}\frac{a_0}{g}$$

EMB/Gg1/08

7.



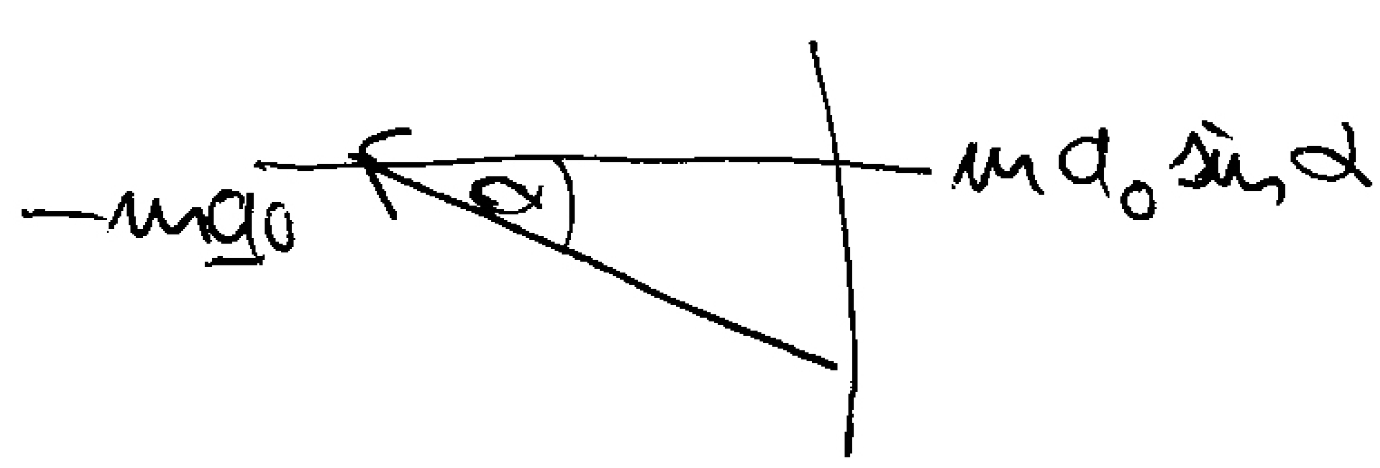
a) $(m\ddot{\underline{r}}')_{\parallel} = (\underline{F}_{rug})_{\parallel} + (m\underline{g})_{\parallel} - $
$- (m\,\underline{a}_0)_{\parallel}$

$(m\ddot{r}')_{\parallel} = m\ddot{y}'$

$(\underline{F}_{rug})_{\parallel} = -ky'$

$(m\underline{g})_{\parallel} = -mg$

$(-m\underline{a}_0)_{\parallel} = m\,a_0 \sin\alpha$



$\Rightarrow$

$m\ddot{y}' = -ky' - mg + m a_0 \sin\alpha$

b)
$\ddot{y}' \overset{!}{=} 0$

$\Rightarrow$

$0 = -ky'^{*} - mg +$
$+ m a_0 \sin\alpha$

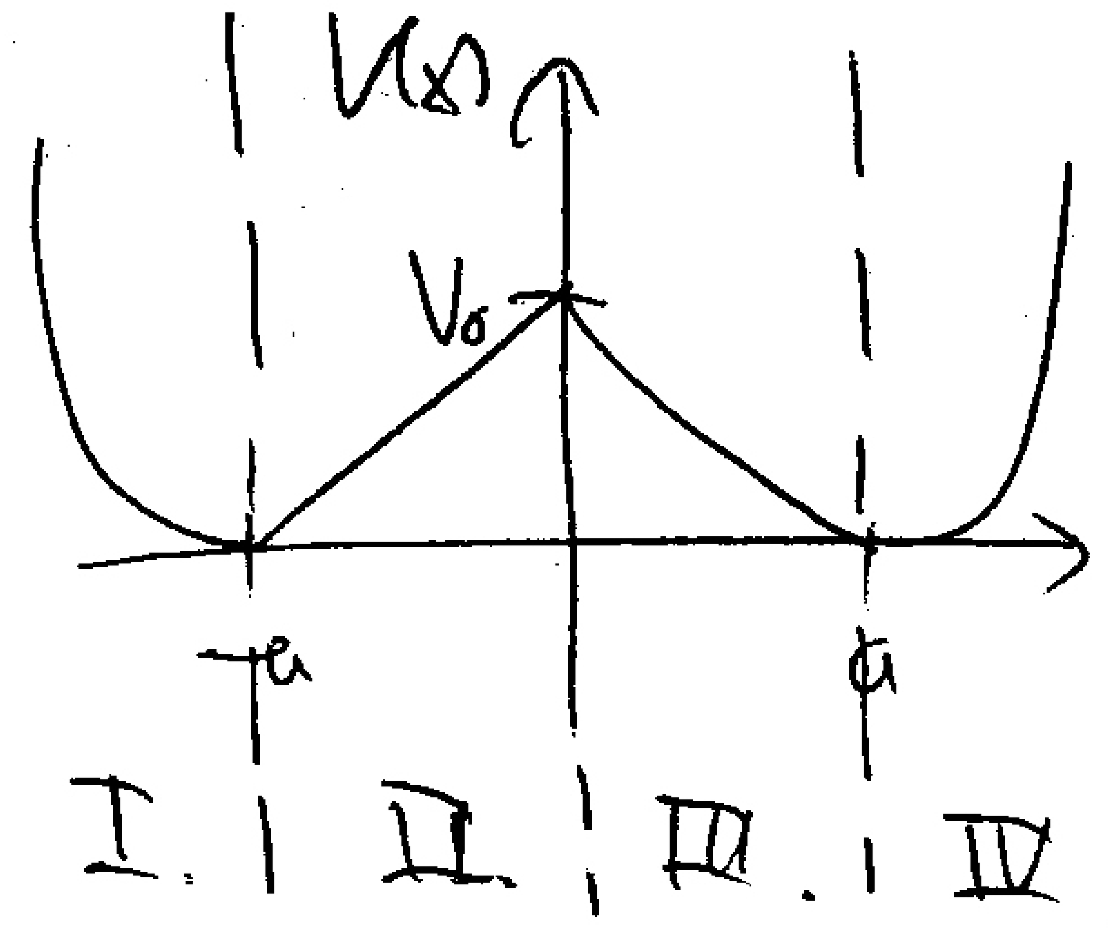
$y'^{*} \overset{!}{=} 0 \Rightarrow a_0 \overset{!}{=} ?$

$\Rightarrow 0 \overset{!}{=} -\cancel{m}g + \cancel{m}a_0 \sin\alpha$

$a_0 \overset{!}{=} \dfrac{g}{\sin\alpha}$

8.

$$V(x)=\begin{cases}\frac{1}{2}k(x+a)^2, & \text{ha } x<-a\\ V_0\left(1-\frac{|x|}{a}\right), & \text{ha } |x|\leq a\\ \frac{1}{2}k(x-a)^2, & \text{ha } x>a\end{cases}$$



• fordulópontok: $E=V(x_{fp})$

I.

$$E=\frac{1}{2}k(x_{fp1}+a)^2$$

$$\rightarrow x_{fp1}=-a-\frac{\sqrt{2E}}{\sqrt{k}}$$

Ez az 1 megó. van a) és b) esetben is, ugyanis $-a+\frac{\sqrt{2E}}{\sqrt{k}}\notin$ I. semmilyen E mellett.

a), b) $\}$ x_{fp1} valóban létezik

II.

$$E=V_0\left(1+\frac{x_{fp2}}{a}\right)$$

$$\rightarrow x_{fp2}=a\left(\frac{E}{V_0}-1\right)$$

a) $x_{fp2}<0$, így $x_{fp2}\in$ II, valóban létezik.

b) $x_{fp2}>0$, így $x_{fp2}\notin$ II, azaz nem valósul meg.

III.

$$E=V_0\left(1-\frac{x_{fp3}}{a}\right)$$

$$\rightarrow x_{fp3}=a\left(1-\frac{E}{V_0}\right)$$

a) $x_{fp3}\in$ III. valóban létezik.

b) $x_{fp3}\notin$ III., így nem létezik.

IV.

$$E=\frac{1}{2}k(x_{fp4}-a)^2$$

$$\rightarrow x_{fp4}=a+\frac{\sqrt{2E}}{\sqrt{k}}$$

Ez az 1 megó. van a) és b) esetben is, ugyanis $a-\frac{\sqrt{2E}}{\sqrt{k}}<a$, így $a-\frac{\sqrt{2E}}{\sqrt{k}}\notin$ IV. semmilyen E mellett.

a) és b) esetben is: x_{fp4} valóban létezik.

- egyenlet:

I. $\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k (x+a)^2 = E$ ellipszis

II. $\frac{1}{2} m \dot{x}^2 + V_0\left(1+\frac{x}{a}\right) = E$ fekvő parabola

III. $\frac{1}{2} m \dot{x}^2 + V_0\left(1-\frac{x}{a}\right) = E$ fekvő parabola

IV. $\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k (x-a)^2 = E$ ellipszis

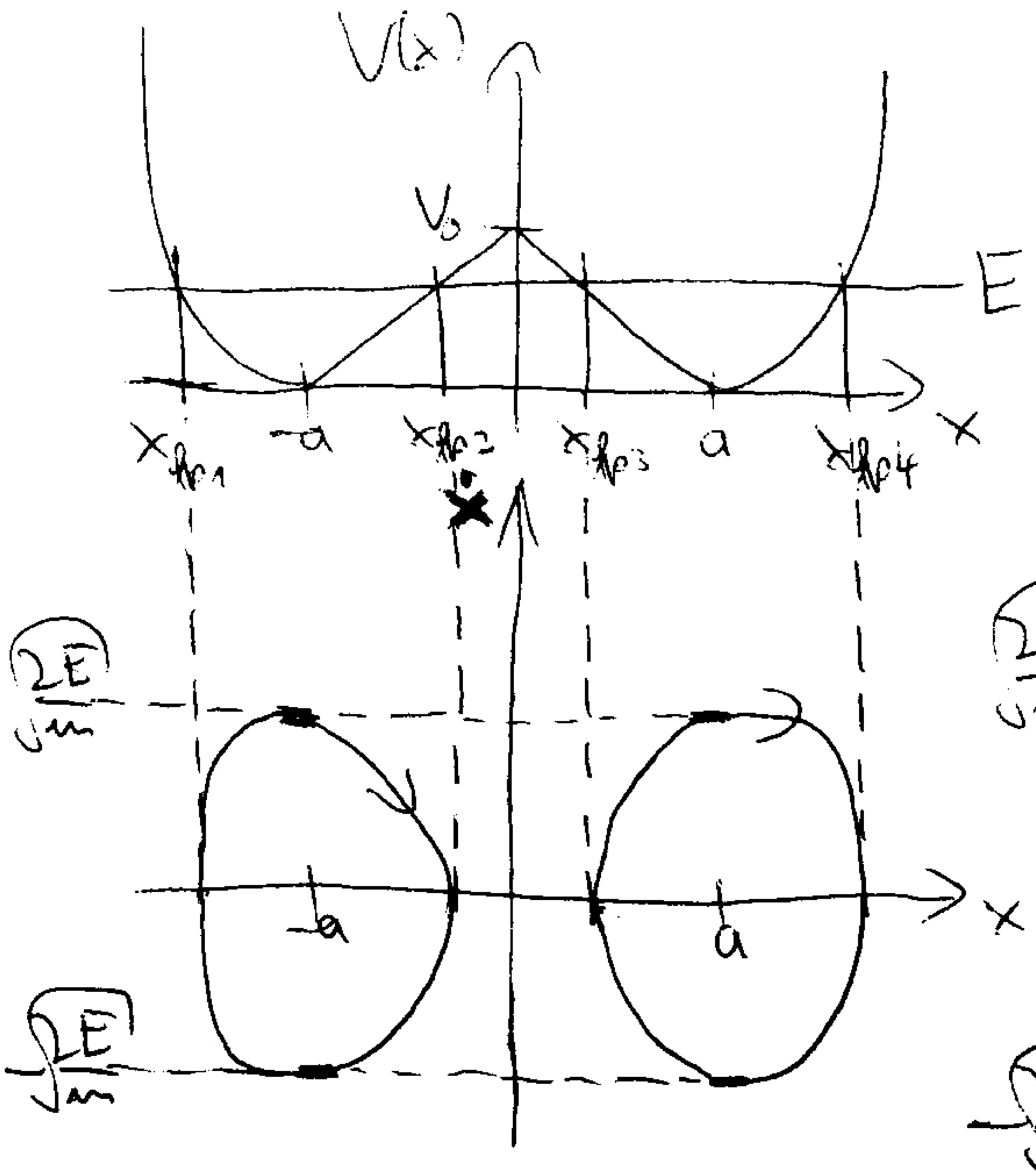
-

$\dot{x}(x=-a) = \pm\sqrt{\frac{2}{m}\left(E - V_0\left(1+\frac{-a}{a}\right)\right)} = \pm\sqrt{\frac{2E}{m}}$ a) és b) esetben is megvalósul.

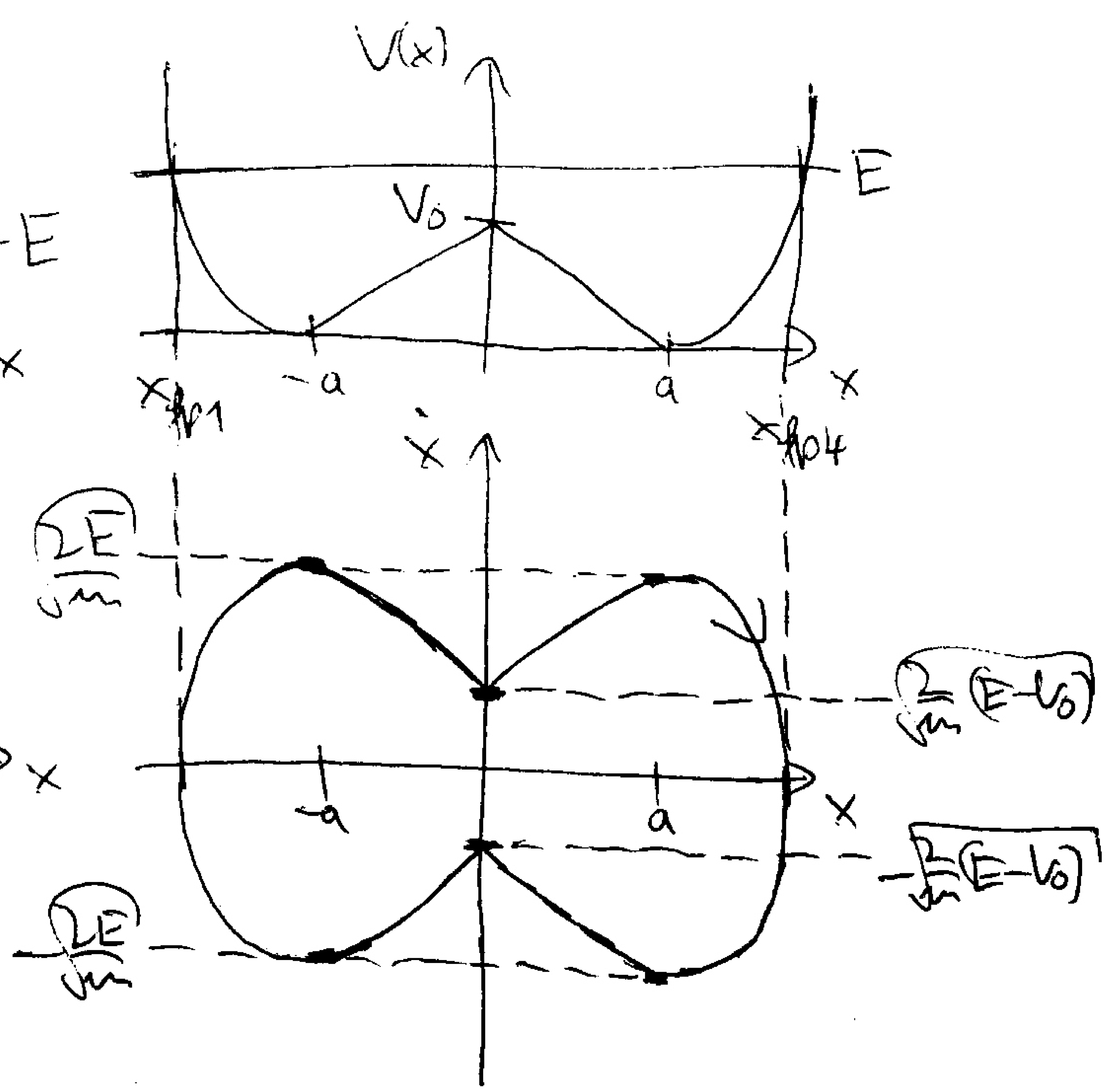
$\dot{x}(x=0) = \pm\sqrt{\frac{2}{m}\left(E - V_0\left(1+\frac{0}{a}\right)\right)} = \pm\sqrt{\frac{2}{m}(E-V_0)}$ csak b) esetben valósul meg!

$\dot{x}(x=a) = \pm\sqrt{\frac{2}{m}\left(E - V_0\left(1-\frac{a}{a}\right)\right)} = \pm\sqrt{\frac{2E}{m}}$ a) és b) esetben is megvalósul.

a)

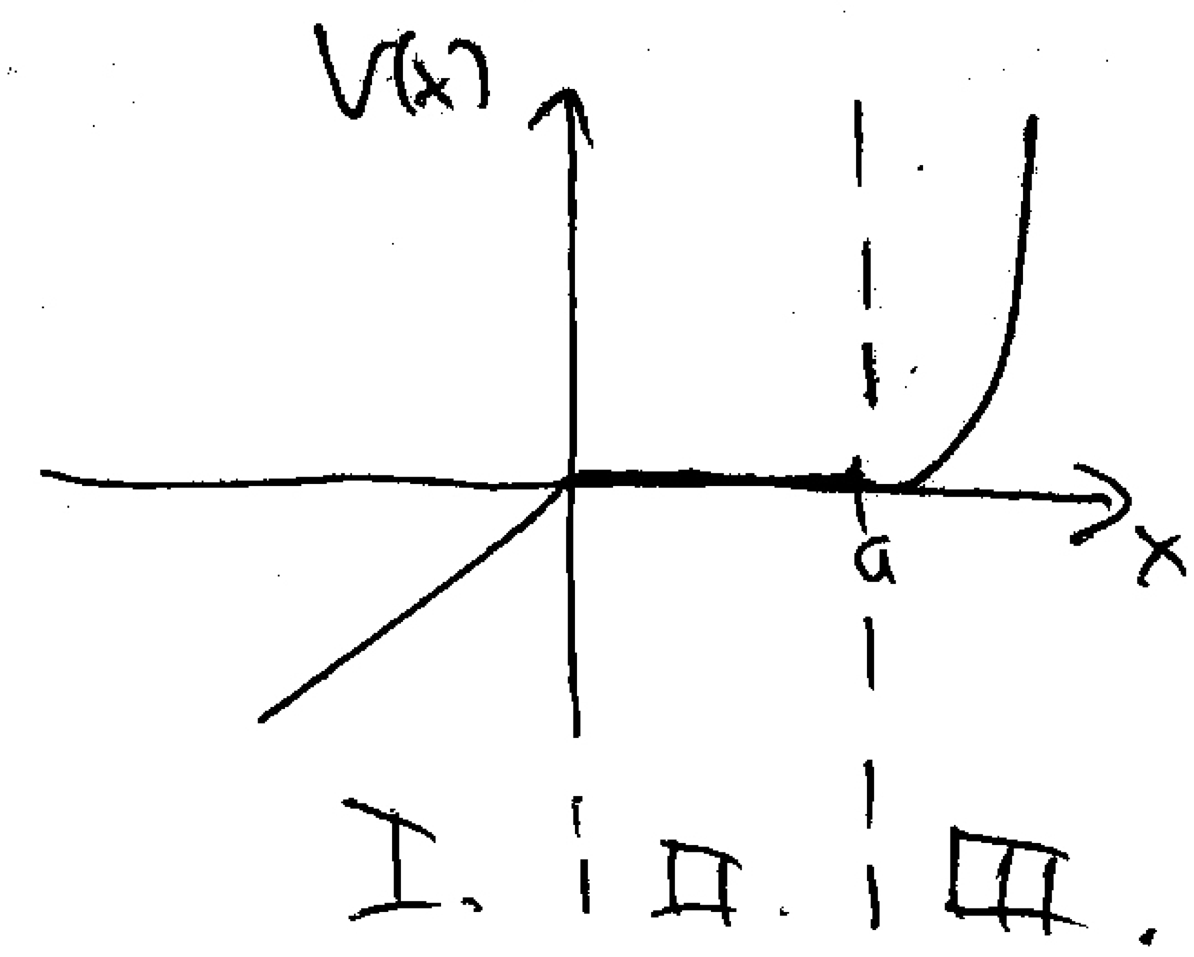


b)



9.

$$V(x)=\begin{cases}\alpha x, & \text{ha } x<0\\ 0, & \text{ha } 0\le x<a\\ \frac{1}{2}\ell(x-a)^2, & \text{ha } x\ge a\end{cases}$$



- fordulópontok: $E=V(x_{fp})$

I.

$E=\alpha x_{fp1}$

$\Rightarrow x_{fp1}=\frac{E}{\alpha}$

a) $x_{fp1}<0$, így $x_{fp1}\in I$, vagyis x_{fp1} létezik.

b) $x_{fp1}>0$, ezért $x_{fp1}\notin I$, így nem valósul meg.

II.

$E=0$

Tehát az $E=0$ feltételt kaptuk arra, hogy a II. tartományban legyen fordulópont. Ha ez teljesül, akkor a II. tartomány minden pontja kielégíti a fordulópontra vonatkozó feltételt. Azonban az $E=0$ eset vizsgálata nem része a feladatnak a szöveg szerint.

III.

$E=\frac{1}{2}\ell(x_{fp2}-a)^2$

$\Rightarrow x_{fp2}=a+\sqrt{\frac{2E}{\ell}}$

Elvileg lenne egy $x_{fp}=a-\sqrt{\frac{2E}{\ell}}$ megoldás is, de $a-\sqrt{\frac{2E}{\ell}}<a$, vagyis $a-\sqrt{\frac{2E}{\ell}}\notin III$. semmilyen E mellett. Így tehát csak $x_{fp2}=a+\sqrt{\frac{2E}{\ell}}$ létezhet.

a) $E<0 \Rightarrow x_{fp2}$ nem valós, ami azt jelenti, hogy x_{fp2} nem létezik.

b) $E>0 \Rightarrow x_{fp2}$ valós, Mivel $x_{fp2}\in III.$, x_{fp2} valóban létezik.

• egyenlet:

I. $\frac{1}{2}m\dot{x}^2 + \alpha x = E$ fekvő parabola

II. $\frac{1}{2}m\dot{x}^2 = E$, átrendezve: $\dot{x} = \pm\sqrt{\frac{2E}{m}}$

Tehát a II. intervallumban az $\dot{x}(x)$ függvény konstans, azaz vízszintes egyenes.

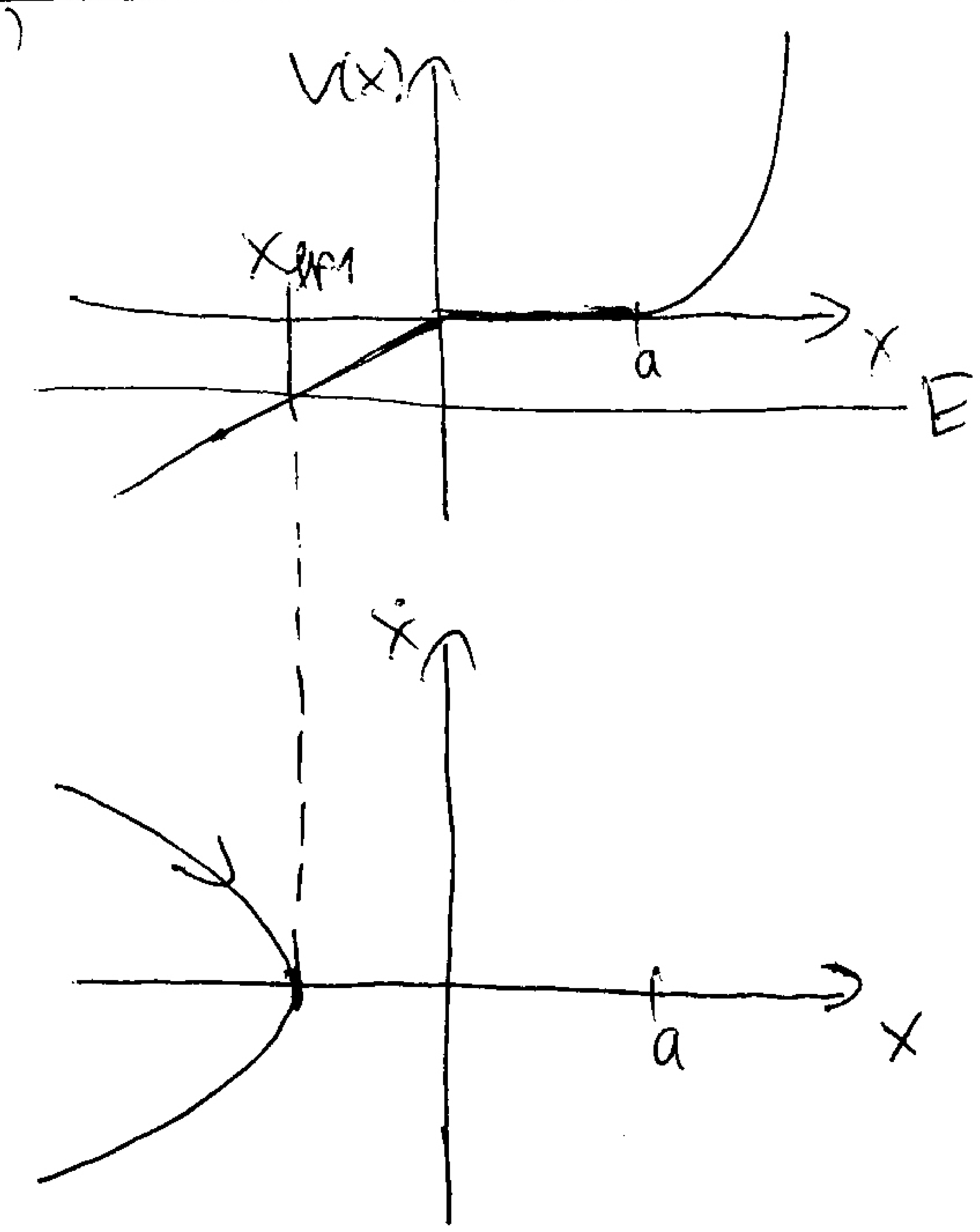
III. $\frac{1}{2}m\dot{x}^2 + \frac{1}{2}k(x-a)^2 = E$ ellipszis

• $\dot{x}(x=0) = \pm\sqrt{\frac{2}{m}(E-0)} = \pm\sqrt{\frac{2E}{m}}$ (Mivel $0 \in$ II., a fázistérbeli trajektória egyenlete alapján ezt már vártuk.)

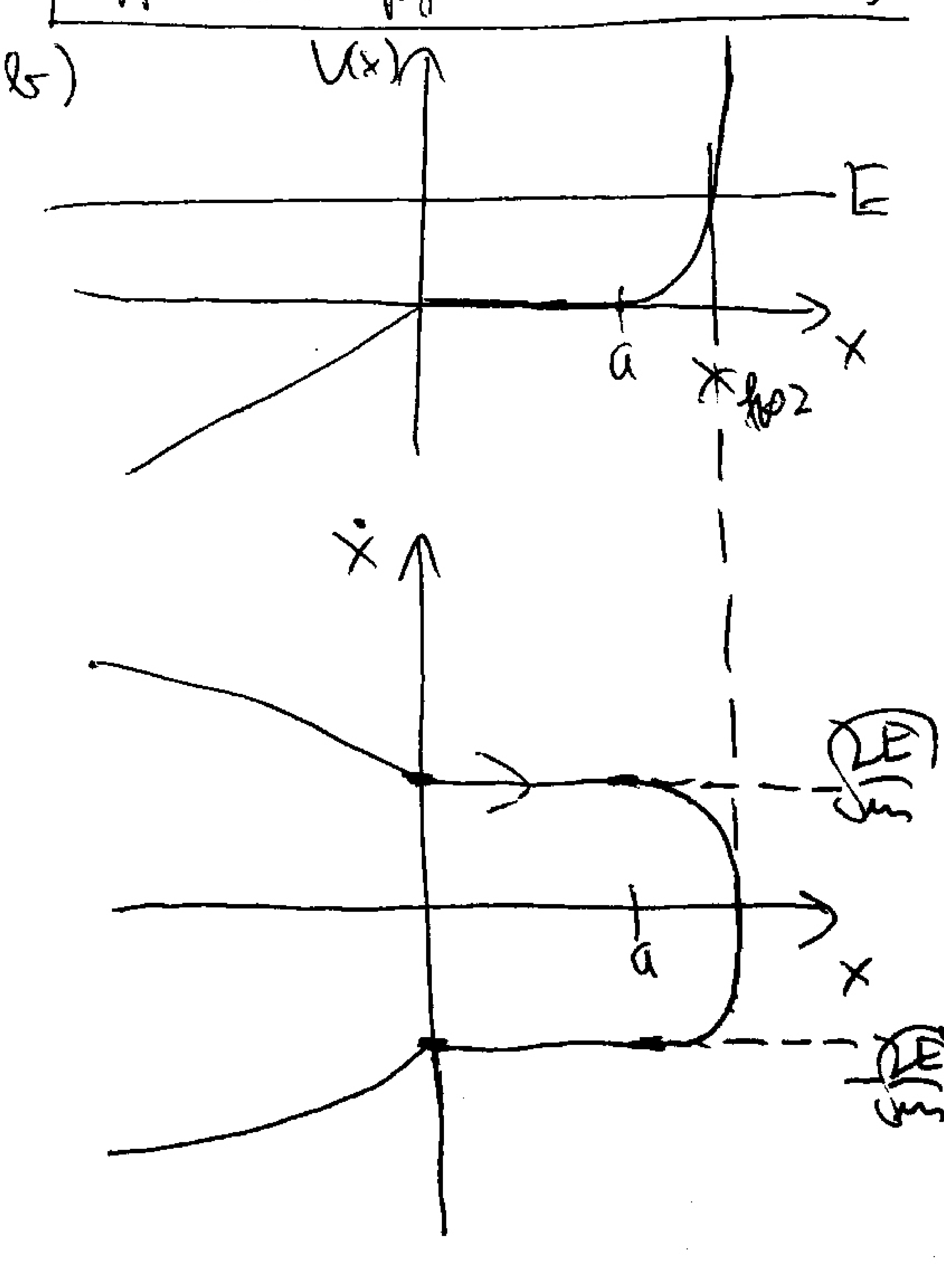
$\dot{x}(x=a) = \pm\sqrt{\frac{2}{m}\left(E - \frac{1}{2}k(a-a)^2\right)} = \pm\sqrt{\frac{2}{m}(E-0)} = \pm\sqrt{\frac{2E}{m}}$ (Mivel $V(x)$ folytonos, és $x=a$ a II. intervallum szélén helyezkedik el, a fázistérbeli trajektória egyenlete alapján ezt is vártuk.)

Mindkét kiszámított sebesség csak b) esetben valósul meg!

a)



b)



$\boxed{10.}$

$$V(x)=\begin{cases}-\alpha x - V_0 & ,\ \text{ha } x\le 0\\ -V_0\cos\frac{\pi x}{2a} & ,\ \text{ha } 0<x<a\\ \frac{1}{2}k(x-a)^2 & ,\ \text{ha } x>a\end{cases}\qquad \alpha, V_0, a, k>0$$

$$-V_0<E<0$$

- fp.:

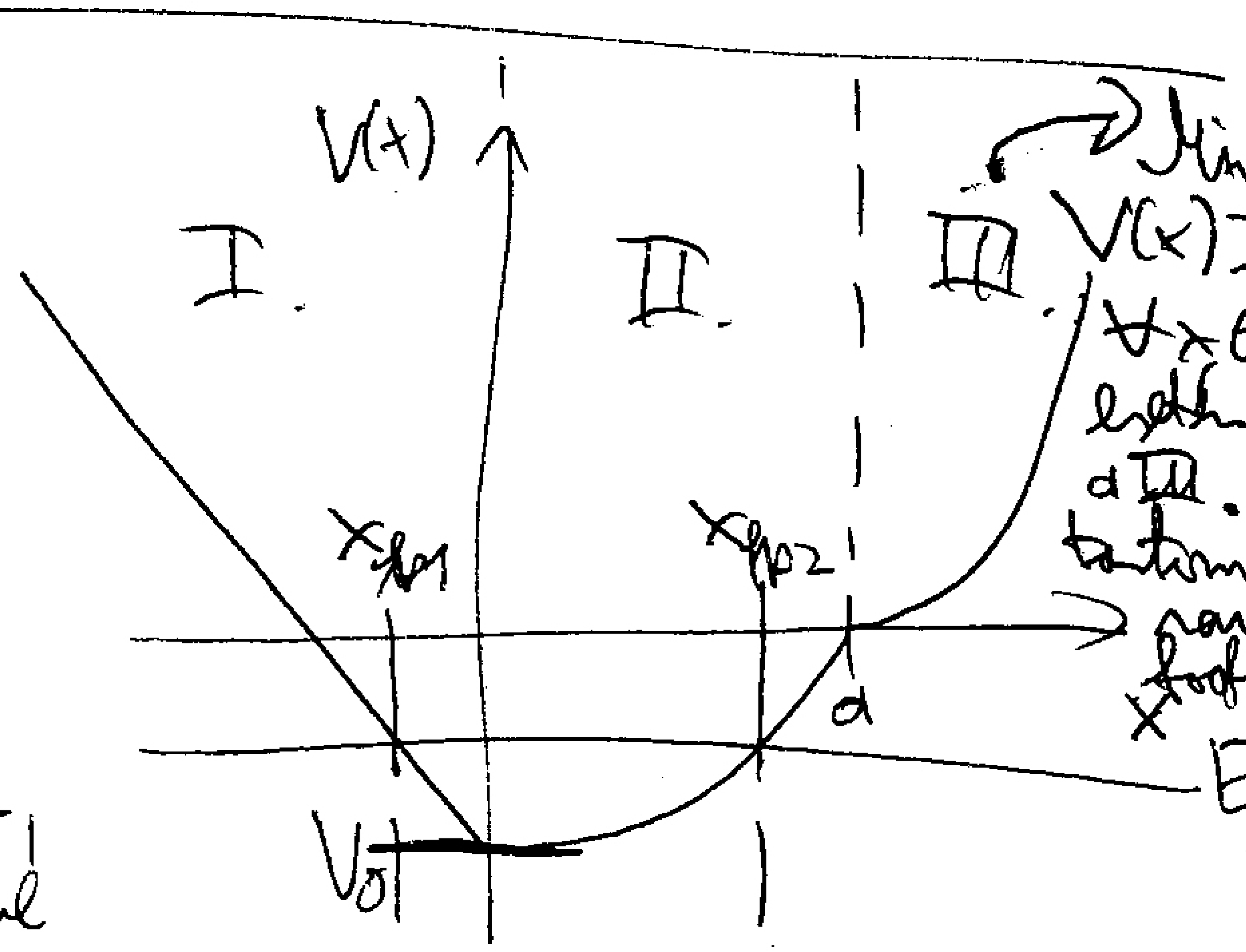
I. $E=V(x_{fp1})$

$E=-\alpha x_{fp1}-V_0$

$x_{fp1}=-\dfrac{V_0+E}{\alpha}$

$x_{fp1}<0 \Rightarrow x_{fp1}\in I.$!

Így x_{fp1} megoldás!



Mivel $V(x)>E$ $\forall x\in III.$, ezért a III. tartománnyal nem foglalkozunk.

II. $E=V(x_{fp2})$

$E=-V_0\cos\dfrac{\pi x_{fp2}}{2a}$

$\dfrac{\pi x_{fp2}}{2a}=\arccos\left(-\dfrac{E}{V_0}\right)$

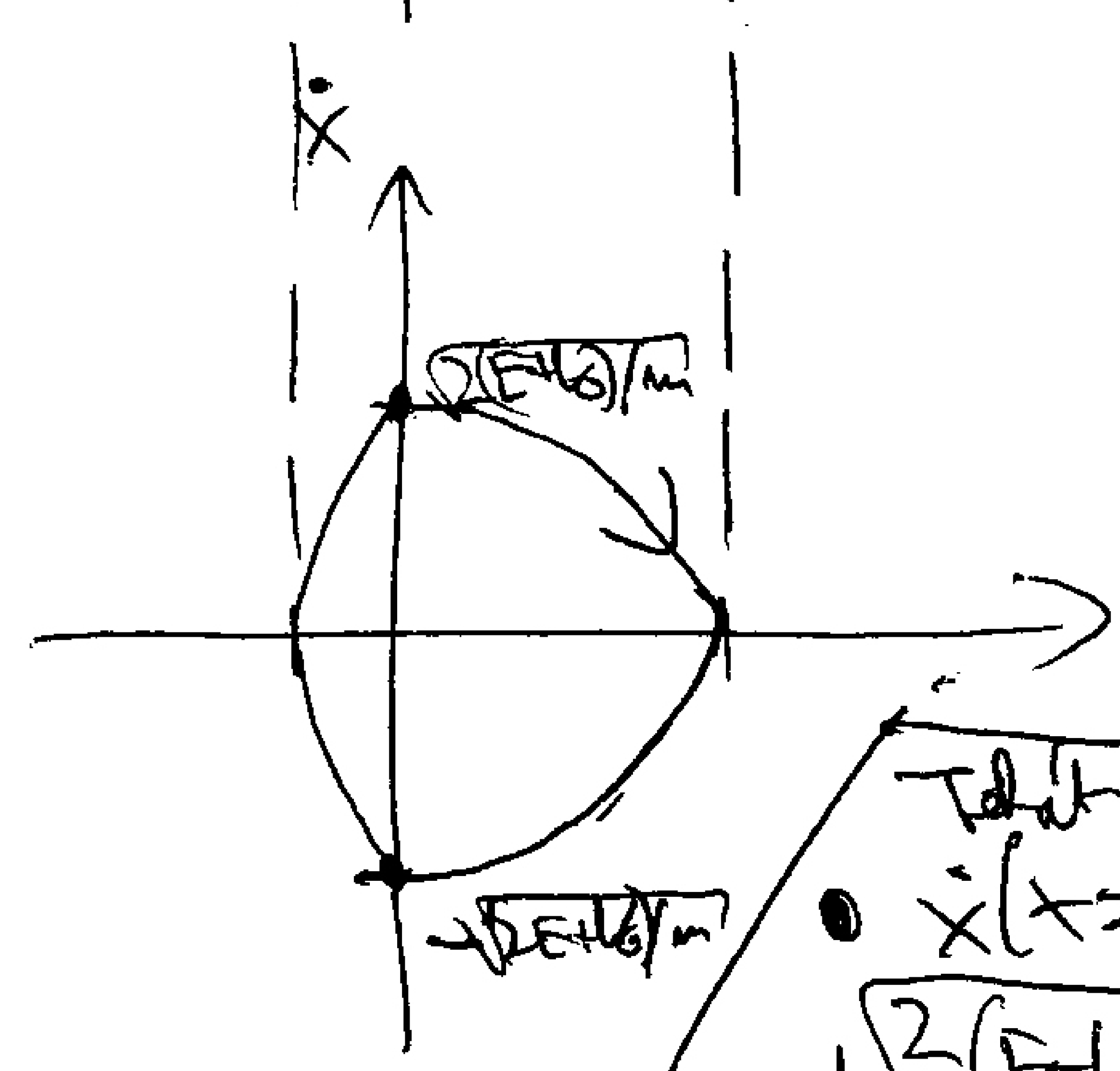
$x_{fp2}=\dfrac{2a}{\pi}\arccos\left(-\dfrac{E}{V_0}\right)$

$E<0 \Rightarrow \arccos\left(-\frac{E}{V_0}\right)<\frac{\pi}{2}$

$\Rightarrow \frac{2a}{\pi}\arccos\left(-\frac{E}{V_0}\right)<\frac{2a}{\pi}\frac{\pi}{2}=a$

Ugyanakkor nyilván $x_{fp}>0$.

Így $x_{fp2}\in II.$, x_{fp2} megoldás.



Tehát megoldás.

- $\dot{x}(x=0)=$
$=\pm\sqrt{\frac{2}{m}(E-V(x=0))}=$
$=\pm\sqrt{\frac{2}{m}(E+V_0)}\in\mathbb{R}$,
mivel $E+V_0>0$.

- egyenlet: I. $E=$ ~~[illegible]~~ $-\alpha x-V_0+\frac{1}{2}m\dot{x}^2$ Csak felső parabola lehet!

II. $E=-V_0\cos\frac{\pi x}{a}+\frac{1}{2}m\dot{x}^2$

11.

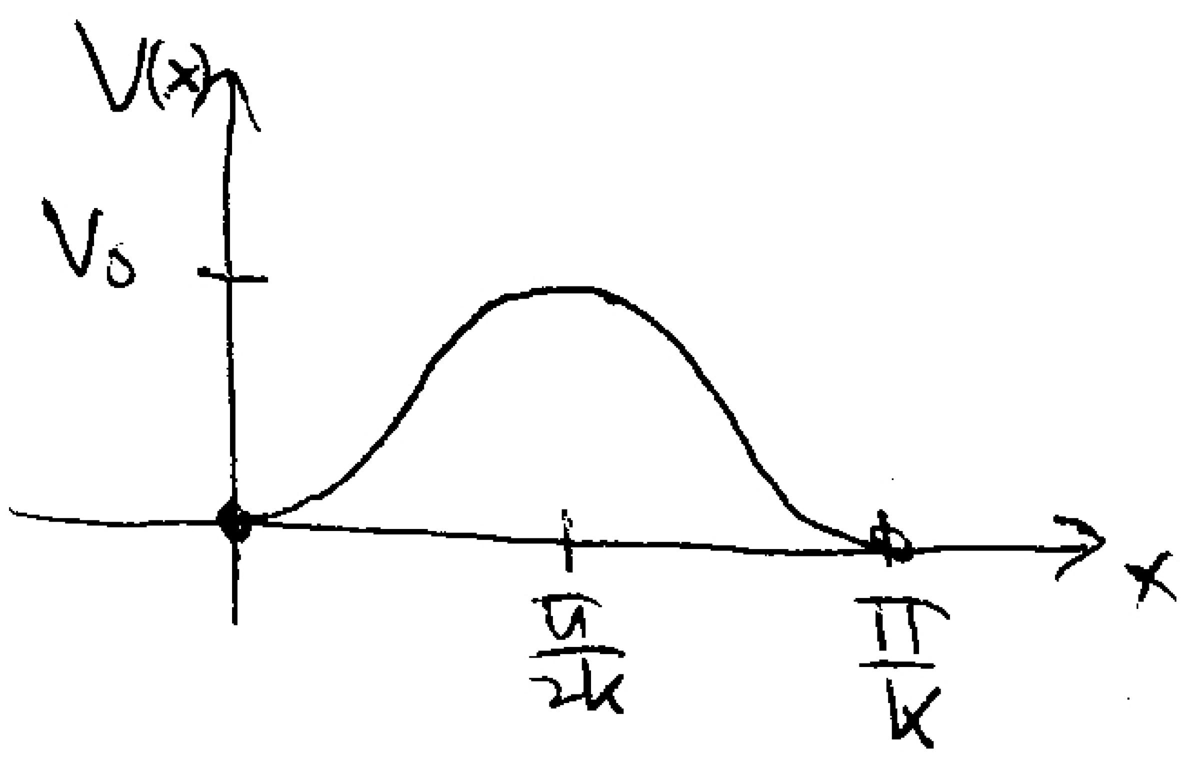
$$V(x) = V_0 \sin^2(Kx) = V_0 \frac{1-\cos(2Kx)}{2}$$

$$\begin{aligned}\cos(2\varphi) &= \cos^2\varphi - \sin^2\varphi = \\ &= (1-\sin^2\varphi) - \sin^2\varphi = \\ &= 1 - 2\sin^2\varphi \\ \Rightarrow \sin^2\varphi &= \frac{1-\cos(2\varphi)}{2}\end{aligned}$$

A cos „hullámhossza": $\frac{2\pi}{2K} = \frac{\pi}{K}$, vagyis éppen az értelmezési tartomány végére fejeződik be 1 periódus.

$\Rightarrow$

HA $V_0 > 0$ és $K > 0$



Ennyi tájékozódást érdemes megtenni, azonban ez elvileg nem része a feladat megoldásának.

$$V'(x) = V_0 \cdot 2\sin(Kx)\cdot\cos(Kx)\cdot K = V_0 \cdot K \cdot \sin(2Kx)$$

$$V'(x)\big|_{x=x^*} = V_0 \cdot K \cdot \sin(2Kx^*) = 0$$

$$\Rightarrow \quad \sin(2Kx^*) = 0$$

$2Kx_1^* = 2\pi m_1, \; m_1 \in \mathbb{Z}$

$x_1^* = \frac{2\pi m_1}{2K} = \frac{\pi}{K} m_1$

Mivel $x \in \left[0, \frac{\pi}{K}\right)$, csak az $m_1 = 0$ lesz jó:

$x_1^* = 0$

$2Kx_2^* = \pi + 2\pi m_2, \; m_2 \in \mathbb{Z}$

$x_2^* = \frac{\pi + 2\pi m_2}{2K} = \frac{\pi}{2K} + \frac{\pi}{K} m_2$

Csak $m_2 = 0$ jó:

$x_2^* = \frac{\pi}{2K}$

$$V''(x) = V_0 K \cos(2Kx)\cdot 2K = 2V_0 K^2 \cos(2Kx)$$

$$x_1^*: \quad V''(x)\big|_{x=x_1^*} = 2V_0k^2 \underbrace{\cos(2k\cdot 0)}_{=1} = 2V_0k^2$$

$2V_0k^2 > 0$ akkor és csak akkor, ha $V_0 > 0$. Ebben az esetben lesz x_1^* stabil.

Ha x_1^* stabil, akkor a kis rezgések körfrekvenciája formálisan:

$$\omega_1 = \sqrt{\frac{1}{m} V''(x)\big|_{x=x_1^*}} = \sqrt{\frac{2V_0k^2}{m}} = |k|\sqrt{\frac{2V_0}{m}}$$

De x_1^* az értelmezési tartomány szélén van, ezért a kis rezgések valójában kilógnak a potenciál értelmezési tartományából. Ha az értelmezési tartományt kiterjesztjük, csak ebben az esetben valósulnak meg ténylegesen a kis rezgések.

$$x_2^*: \quad V''(x)\big|_{x=x_2^*} = 2V_0k^2 \underbrace{\cos\left(2k\cdot\frac{\pi}{2k}\right)}_{=-1} = -2V_0k^2$$

$-2V_0k^2 > 0$ akkor és csak akkor, ha $V_0 < 0$. Ekkor x_2^* stabil.

Ha x_2^* stabil:

$$\omega_2 = \sqrt{\frac{1}{m} V''(x)\big|_{x=x_2^*}} = \sqrt{-\frac{2V_0k^2}{m}} = |k|\cdot\sqrt{-\frac{2V_0}{m}}$$

12.

$$V'(x) = V_0\left(-\frac{a}{(x+a)^2} + \frac{a}{(x-a)^2}\right)$$

$$V'(x)\big|_{x=x^*} = V_0 a\left(-\frac{1}{(x^*+a)^2} + \frac{1}{(x^*-a)^2}\right) = 0$$

$$\Rightarrow \quad -\frac{1}{(x^*+a)^2} + \frac{1}{(x^*-a)^2} = 0$$

$$(x^*-a)^2 = (x^*+a)^2$$

$x^*-a = x^*+a$
$-a = a$ ↯

$x^*-a = -(x^*+a)$

$$x^* = 0 \longleftarrow 2x^* = 0$$

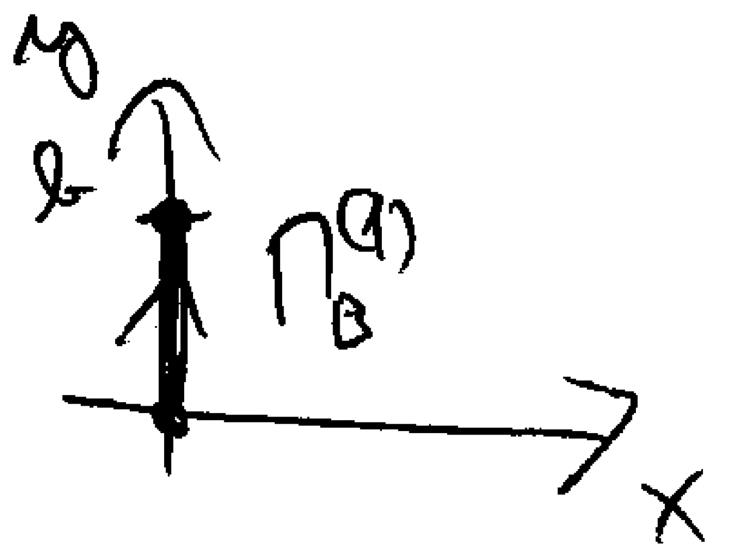
$$V''(x) = V_0\left(2\frac{a}{(x+a)^3} - 2\frac{a}{(x-a)^3}\right) = 2V_0 a\left(\frac{1}{(x+a)^3} - \frac{1}{(x-a)^3}\right)$$

$$V''(x)\Big|_{x=x^*} = 2V_0 a\left(\frac{1}{(0+a)^3} - \frac{1}{(0-a)^3}\right) = 2V_0 a\left(\frac{1}{a^3} + \frac{1}{a^3}\right) =$$

$$= 4V_0\frac{a}{a^3} = 4\frac{V_0}{a^2} > 0 \Rightarrow$$ x^* körül [illegible] végez.

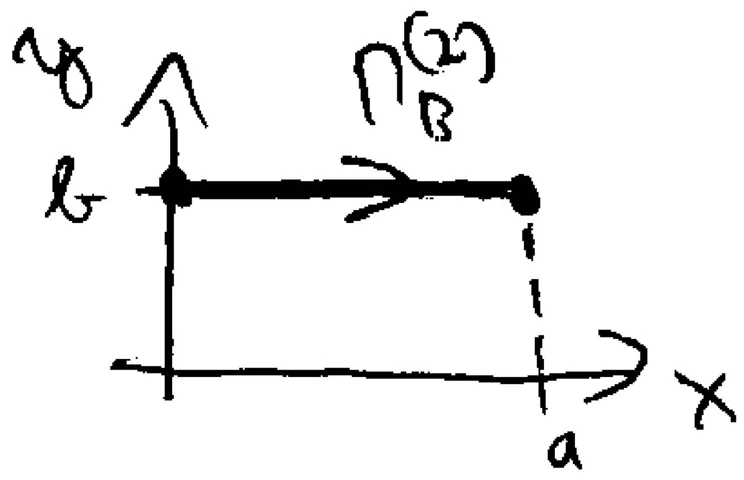
$$\omega = \sqrt{\frac{1}{m}V''(x)\Big|_{x=x^*}} = \sqrt{\frac{1}{m}4\frac{V_0}{a^2}} = \quad 2\frac{1}{a}\sqrt{\frac{V_0}{m}}$$

13.



$$\Gamma_B^{(1)}:\ l=b,\ x=0,\ y=s,\ \underline{T}=\begin{pmatrix}0\\1\end{pmatrix}$$

$$W_{\Gamma_B^{(1)}}=\int_0^b C\cdot 0^2\cdot s\cdot\frac{1}{\sqrt{2}}(-1\ \ 1)\begin{pmatrix}0\\1\end{pmatrix}ds=0$$



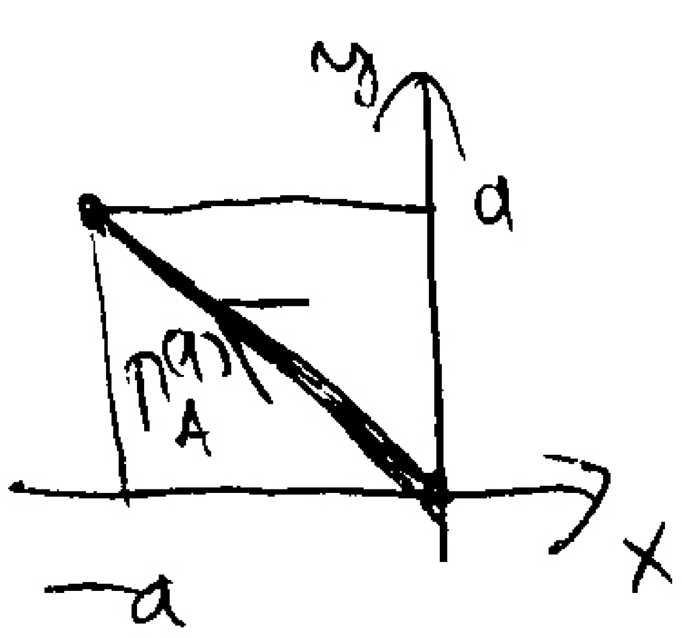
$$\Gamma_B^{(2)}:\ l=a,\ x=s,\ y=b,\ \underline{T}=\begin{pmatrix}1\\0\end{pmatrix}$$

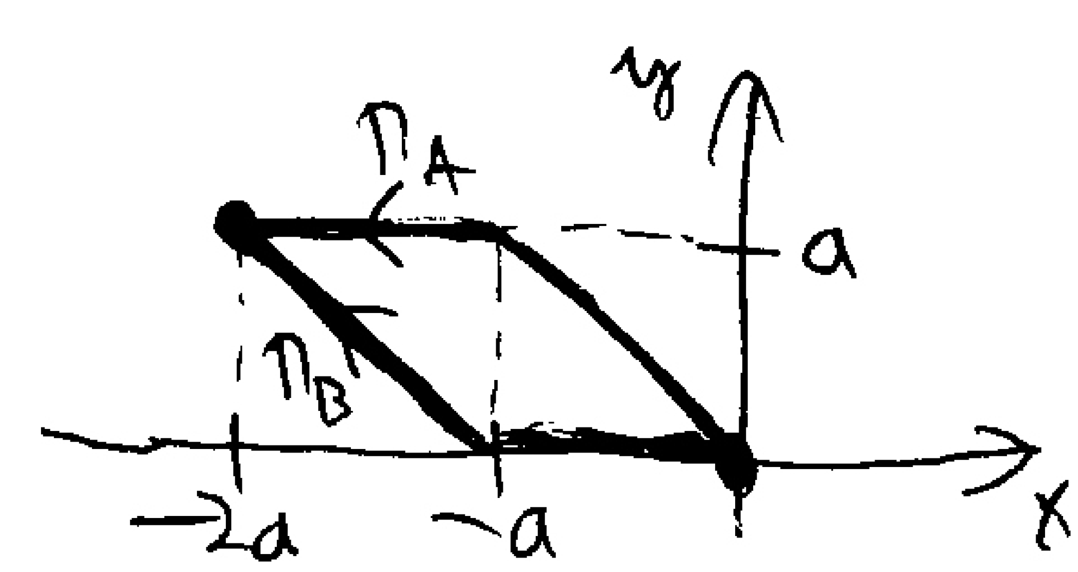
$$W_{\Gamma_B^{(2)}}=\int_0^a C\cdot s^2\cdot b\cdot\frac{1}{\sqrt{2}}\underbrace{(-1\ \ 1)\begin{pmatrix}1\\0\end{pmatrix}}_{=-1\cdot1+1\cdot0=-1}ds=$$

$$=-\frac{1}{\sqrt{2}}Cb\int_0^a s^2\,ds=-\frac{1}{3\cdot\sqrt{2}}Ca^3b$$

$$W_{\Gamma_B}=W_{\Gamma_B^{(1)}}+W_{\Gamma_B^{(2)}}=0-\frac{1}{3\cdot\sqrt{2}}Ca^3b=-\frac{1}{3\sqrt{2}}Ca^3b$$

14.





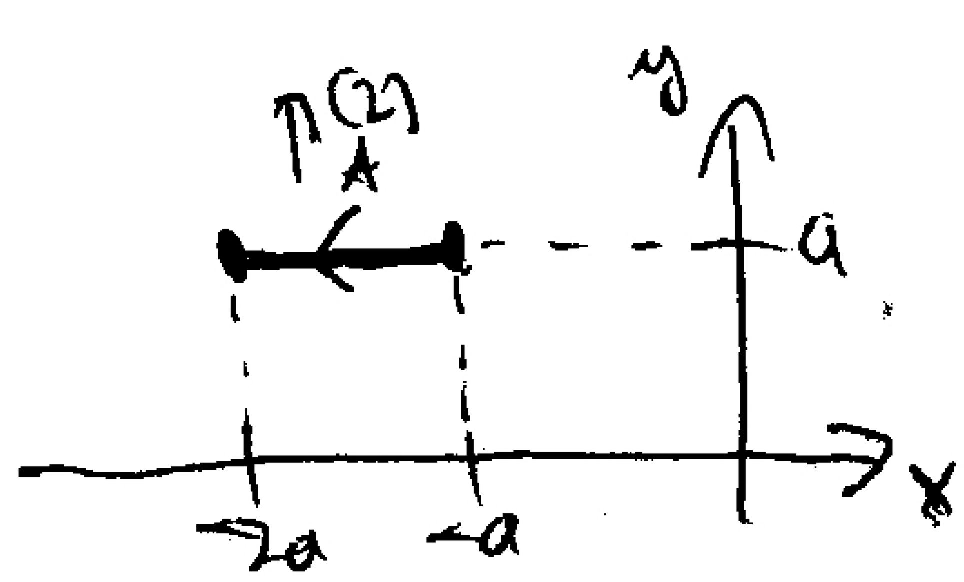
$$\Gamma_A^{(1)}:\ l=\sqrt{2}a$$

$$x=-s\cdot\cos45^\circ=-\frac{s}{\sqrt{2}}$$

$$y=+s\cdot\sin45^\circ=\frac{s}{\sqrt{2}}$$

$$\underline{T}=\frac{d}{ds}\begin{pmatrix}x(s)\\y(s)\end{pmatrix}=\frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix}$$

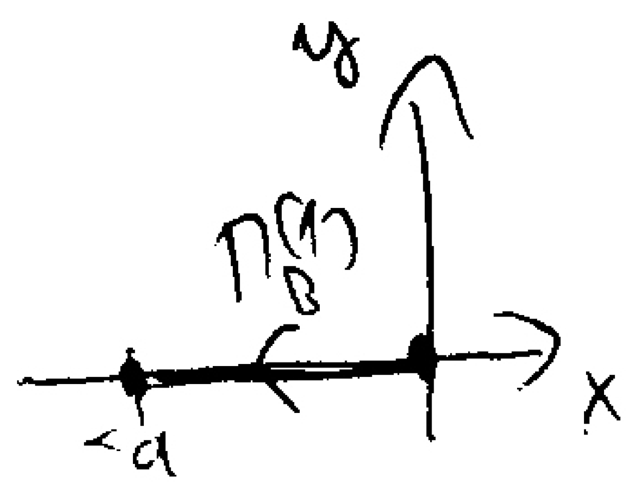
$$W_{\Gamma_A^{(1)}}=\int_0^{\sqrt{2}a}C\cdot\left(\frac{s}{\sqrt{2}}\right)^2\cdot\left(\frac{s}{\sqrt{2}}\right)^2\underbrace{(1\ \ 1)\begin{pmatrix}-1\\1\end{pmatrix}}_{=1\cdot(-1)+1\cdot1=0}ds=0$$



$$\Gamma_A^{(2)}:\ l=a,\ x=-a-s,\ y=a,\ \underline{T}=\begin{pmatrix}-1\\0\end{pmatrix}$$

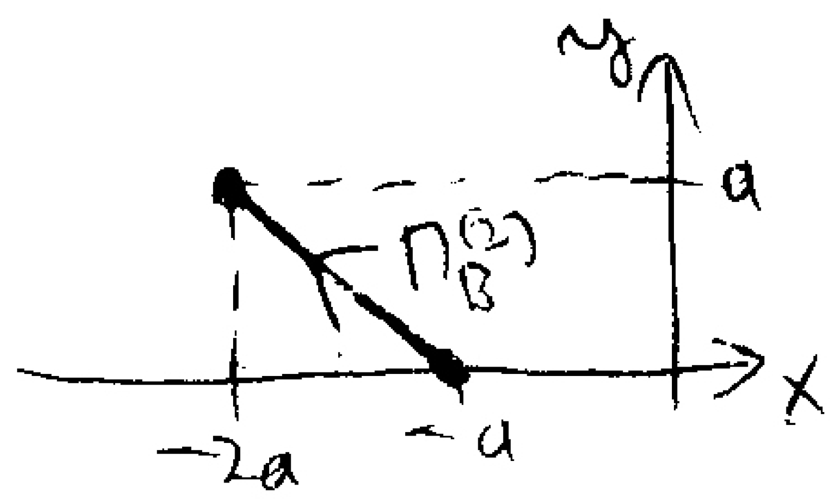
$$W_{\Gamma_A^{(2)}} = \int_0^a C\cdot a^2 \cdot \frac{1}{\sqrt{2}} \underbrace{(1\ \ 1)\begin{pmatrix}-1\\0\end{pmatrix}}_{1\cdot(-1)+1\cdot 0=-1} ds = -\frac{1}{\sqrt{2}}Ca^2\int_0^a 1\,ds = -\frac{1}{\sqrt{2}}Ca^3$$

$$W_{\Gamma_A} = W_{\Gamma_A^{(1)}} + W_{\Gamma_A^{(2)}} = 0 - 1/\sqrt{2}\,Ca^3 = -\frac{1}{\sqrt{2}}Ca^3$$



$\Gamma_B^{(1)}$: $l = a$, $x = -s$, $y = 0$, $\underline{t} = \begin{pmatrix}-1\\0\end{pmatrix}$

$$W_{\Gamma_B^{(1)}} = \int_0^a C\cdot 0^2\, \frac{1}{\sqrt{2}}(1\ \ 1)\begin{pmatrix}-1\\0\end{pmatrix} ds = 0$$



$\Gamma_B^{(2)}$: $l = \sqrt{2}a$, $x = -a - s\cdot\cos 45° = -a - \frac{s}{\sqrt{2}}$,

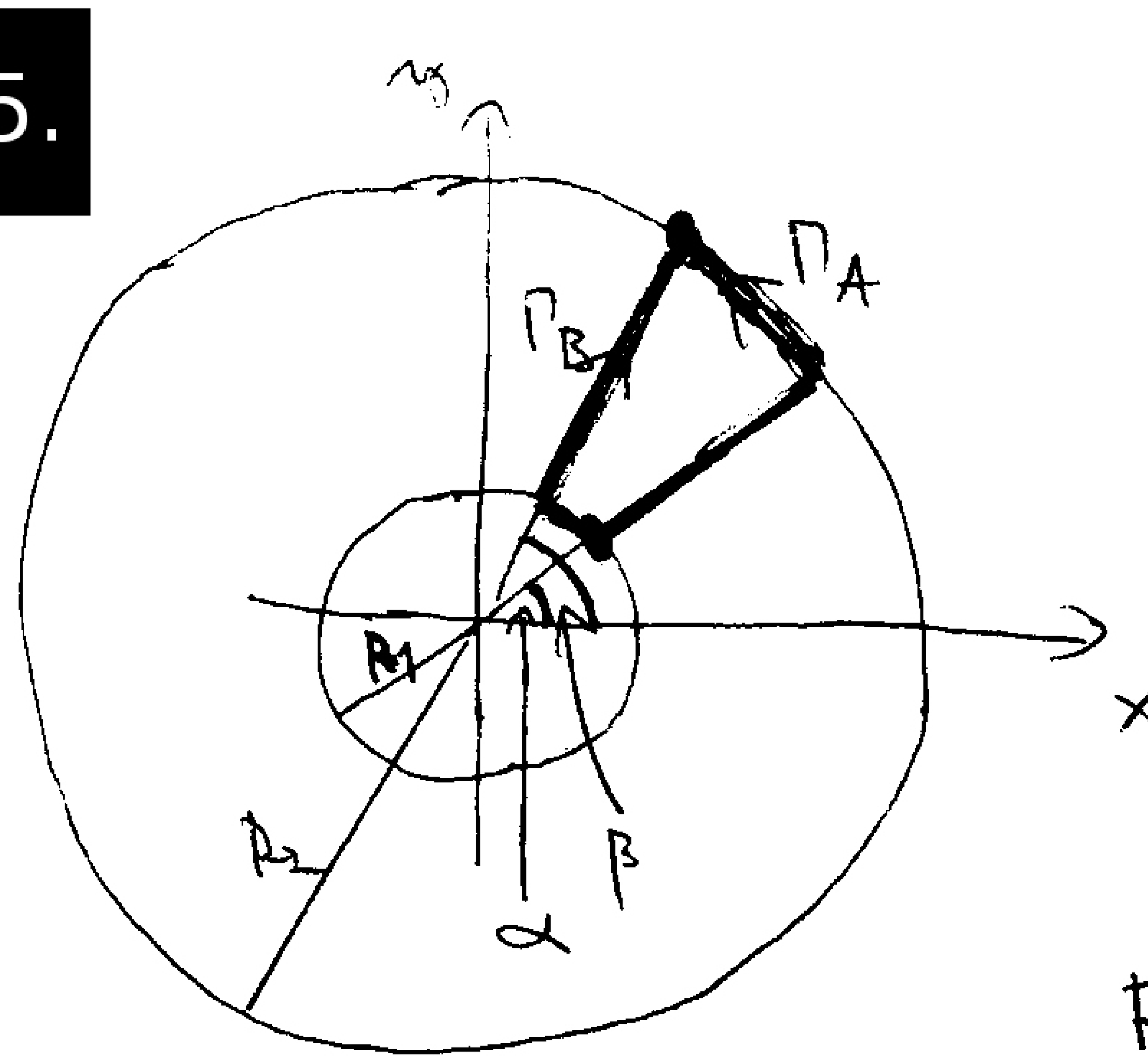
$y = s\cdot\sin 45° = \frac{s}{\sqrt{2}}$, $\underline{t} = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix}$

$$W_{\Gamma_B^{(2)}} = \int_0^{\sqrt{2}a} C\cdot(s/\sqrt{2})^2\left(\frac{1}{\sqrt{2}}\right)^2 \underbrace{(1\ \ 1)\begin{pmatrix}-1\\1\end{pmatrix}}_{1\cdot(-1)+1\cdot 1=0} ds = 0$$

$$W_{\Gamma_B} = W_{\Gamma_B^{(1)}} + W_{\Gamma_B^{(2)}} = 0+0 = 0$$

$W_{\Gamma_B} \neq W_{\Gamma_A} \Rightarrow$ Az erőtér nem lehet konzervatív.

15.



Az erőtér képletét érdemes az

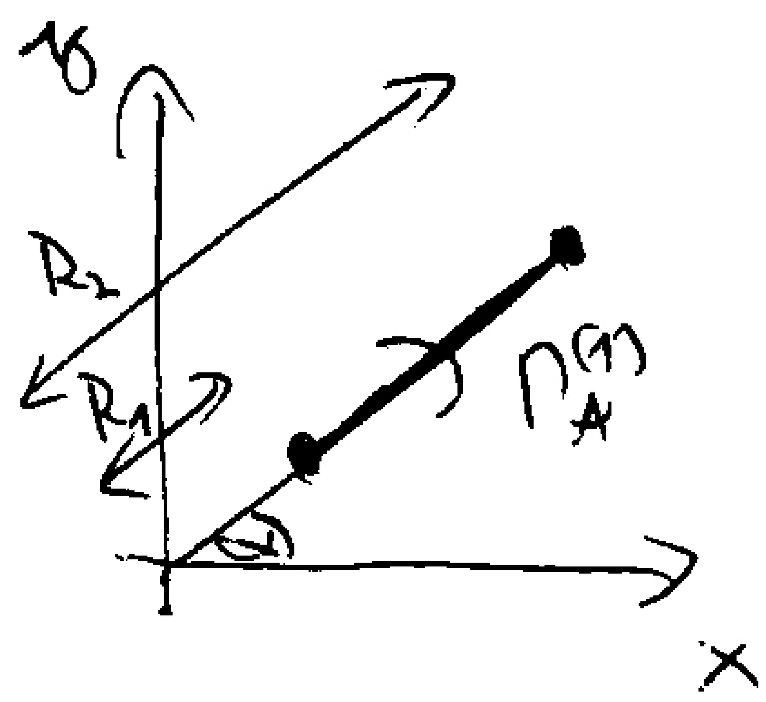
$\underline{r} = r\,\underline{e}_r$

képlet segítségével átalakítani:

$$\underline{F}(\underline{r}) = 4\varepsilon\left[12\left(\frac{\sigma}{r}\right)^{12} - 6\left(\frac{\sigma}{r}\right)^{6}\right]\frac{\cancel{r}\,\underline{e}_r}{r^{\cancel{2}}} =$$

$$= 4\varepsilon\left[12\left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^{6}\right]\frac{1}{r}\underline{e}_r = \Rightarrow$$

$$= 4\varepsilon\left(12\,\frac{\sigma^{12}}{r^{13}} - 6\,\frac{\sigma^{6}}{r^{7}}\right)\underline{e}_r$$



$\Gamma_A^{(1)}$: $\ell = R_2 - R_1$, $r = R_1 + s$, $\varphi = \alpha$, $\underline{I} = \underline{e}_r$

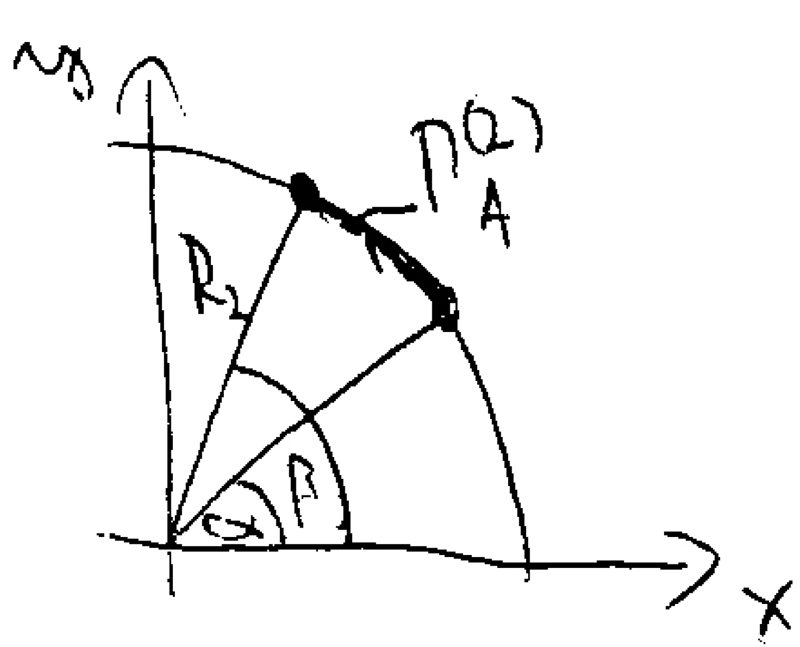
$$W_{\Gamma_A^{(1)}} = \int_0^{R_2 - R_1} 4\varepsilon\left(12\,\frac{\sigma^{12}}{(R_1+s)^{13}} - 6\,\frac{\sigma^{6}}{(R_1+s)^{7}}\right)\underbrace{\underline{e}_r\,\underline{e}_r}_{=1}\,ds =$$

$$= \left[4\varepsilon\left(12\,\sigma^{12}\,\frac{(R_1+s)^{-12}}{-12} - 6\sigma^{6}\,\frac{(R_1+s)^{-6}}{-6}\right)\right]_0^{R_2 - R_1} =$$

$$= 4\varepsilon\left(-\sigma^{12}(\cancel{R_1} + R_2 - \cancel{R_1})^{-12} + \sigma^{6}(\cancel{R_1} + R_2 - \cancel{R_1})^{-6}\right) -$$

$$- 4\varepsilon\left(-\sigma^{12}(R_1 + 0)^{-12} + \sigma^{6}(R_1 + 0)^{-6}\right) =$$

$$= 4\varepsilon\left(-\sigma^{12}\left(\frac{1}{R_2^{12}} - \frac{1}{R_1^{12}}\right) + \sigma^{6}\left(\frac{1}{R_2^{6}} - \frac{1}{R_1^{6}}\right)\right)$$



$\Gamma_A^{(2)}$: $\ell = (\beta - \alpha) R_2$ (körív hossza = szög · sugár),

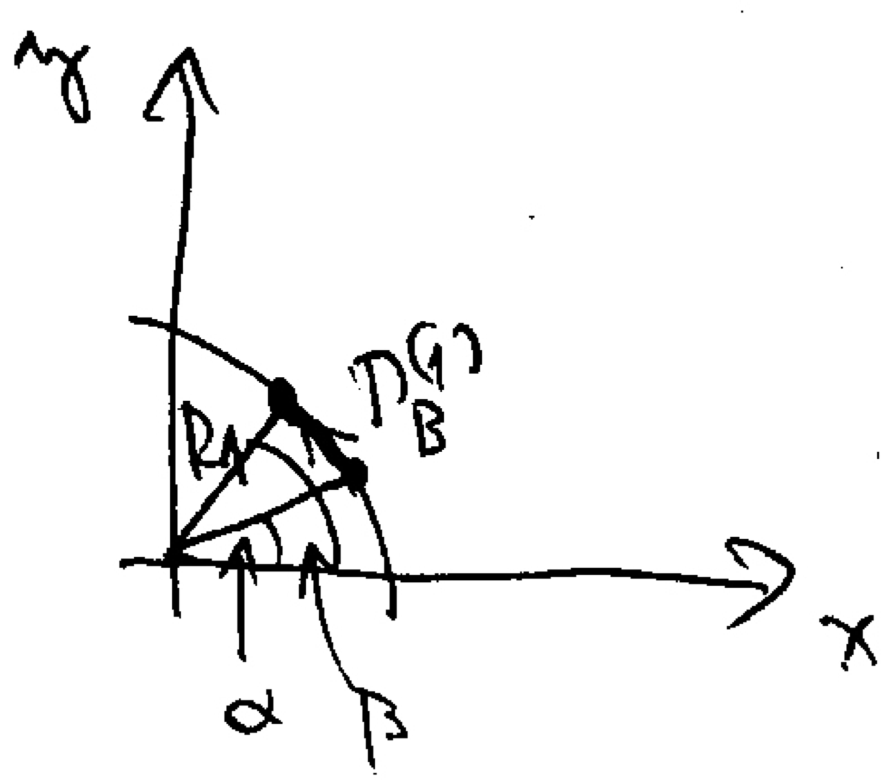
$r = R_2$, $\varphi = \alpha + \frac{s}{R_2}$ $\left(\text{szög} = \frac{\text{körív hossza}}{\text{sugár}}\right)$,

$\underline{I} = \underline{e}_\varphi$

$$W_{\Gamma_A^{(2)}} = \int_0^{(\beta-\alpha)R_2} 4\varepsilon\left(12\,\frac{\sigma^{12}}{R_2^{13}} - 6\,\frac{\sigma^{6}}{R_2^{7}}\right)\underbrace{\underline{e}_r\,\underline{e}_\varphi}_{=0}\,ds = 0$$

$$W_{\Gamma_A} = W_{\Gamma_A^{(1)}} + W_{\Gamma_A^{(2)}} = W_{\Gamma_A^{(1)}} + 0 = W_{\Gamma_A^{(1)}} =$$

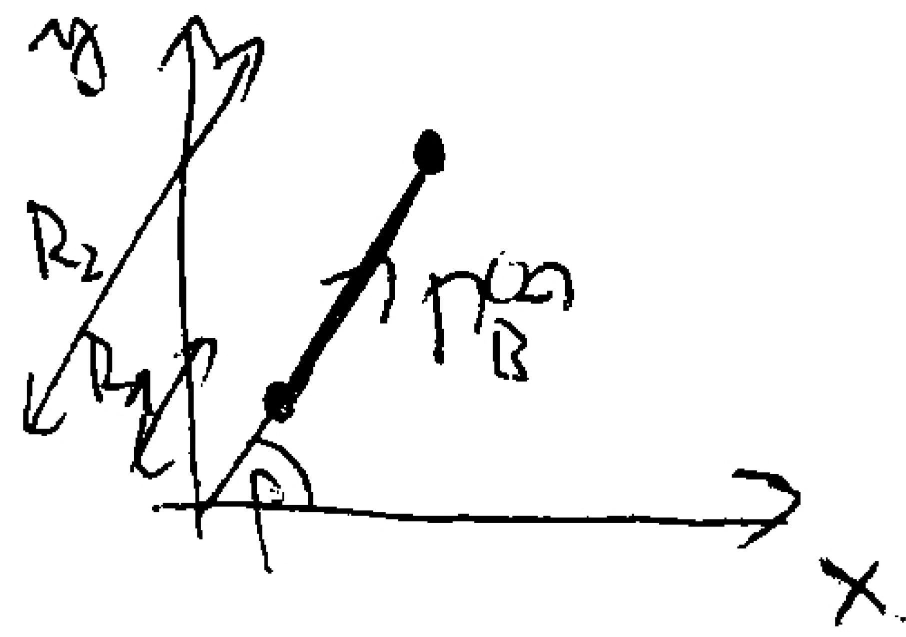
$$= 4\varepsilon\left(\sigma^{6}\left(\frac{1}{R_2^{6}} - \frac{1}{R_1^{6}}\right) - \sigma^{12}\left(\frac{1}{R_2^{12}} - \frac{1}{R_1^{12}}\right)\right)$$



$$\Gamma_B^{(1)}: \ \ell = (\beta-\alpha)\cdot R_1, \ r = R_1, \ \varphi = \alpha + \frac{1}{R_1},$$

$$\frac{\underline{I}}{(\beta-\alpha)R_1} = \underline{e}_\varphi$$

$$W_{\Gamma_B^{(1)}} = \int_0^\ell 4\varepsilon\left(12\frac{\sigma^{12}}{R_1^{13}} - 6\frac{\sigma^6}{R_1^7}\right)\underbrace{\underline{e}_r \underline{e}_\varphi}_{=0}\, ds = 0$$



$$\Gamma_B^{(2)}: \ \ell = R_2 - R_1, \ r = R_1 + s, \ \underbrace{\varphi = \beta}, \ \underline{I} = \underline{e}_r$$

Ettől eltekintve $\Gamma_B^{(2)}$ paraméterezése megegyezik $\Gamma_A^{(1)}$ paraméterezésével.

Mivel φ-t ebben a feladatban nem használjuk fel a munka kiszámítása során:

$$W_{\Gamma_B^{(2)}} = W_{\Gamma_A^{(1)}}$$

$$W_{\Gamma_B} = W_{\Gamma_B^{(1)}} + W_{\Gamma_B^{(2)}} = 0 + W_{\Gamma_A^{(1)}} = W_{\Gamma_A^{(1)}} = W_{\Gamma_A} =$$

$$= 4\varepsilon\left(\sigma^6\left(\frac{1}{R_2^6} - \frac{1}{R_1^6}\right) - \sigma^{12}\left(\frac{1}{R_2^{12}} - \frac{1}{R_1^{12}}\right)\right)$$

$W_{\Gamma_B} = W_{\Gamma_A} \Rightarrow$ lehet potenciál, az erőtér lehet konzervatív.
(Az eredmény alapján nem tudjuk eldönteni, hogy konzervatív-e, ugyanis csak 2 darab görbét vizsgáltunk.)