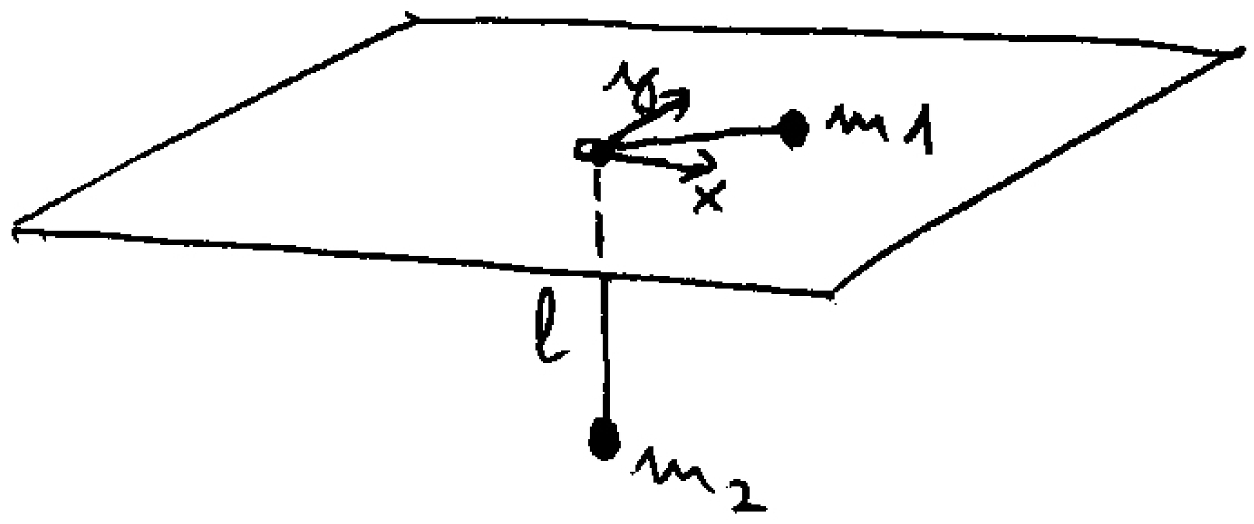
EMB/ZH2/1

1.



1. $x_1, y_1, z_1, x_2, y_2, z_2$
2. $z_1=0,\ x_2=0,\ y_2=0$, kötél: $r-z_2=l$
3. r, φ az (x,y) síkban
4. $x_1 = r\cos\varphi \qquad \dot{x}_1=\dot{r}\cos\varphi - r\sin\varphi\,\dot\varphi$

   $y_1 = r\sin\varphi \qquad \dot{y}_1=\dot{r}\sin\varphi + r\cos\varphi\,\dot\varphi$

   $z_1 = 0 \qquad \dot{z}_1 = 0$

   $x_2 = 0 \qquad \dot{x}_2 = 0$

   $y_2 = 0 \qquad \dot{y}_2 = 0$

   $z_2 = r - l \qquad \dot{z}_2 = \dot{r}$

5 p.

5. $V = m_1 g z_1 + m_2 g z_2 =$

   $= m_2 g r - m_2 g l$

$$T = \frac{1}{2}m_1\left(\dot{x}_1^2+\dot{y}_1^2+\dot{z}_1^2\right)+\frac{1}{2}m_2\left(\dot{x}_2^2+\dot{y}_2^2+\dot{z}_2^2\right)=$$

$$=\frac{1}{2}m_1\left((\dot r\cos\varphi - r\sin\varphi\dot\varphi)^2+(\dot r\sin\varphi + r\cos\varphi\dot\varphi)^2+0\right)+\frac{1}{2}m_2\left(0+0+\dot r^2\right)=$$

$$=\frac{1}{2}m_1\left(\dot r^2+r^2\dot\varphi^2\right)+\frac{1}{2}m_2\dot r^2 = \frac{1}{2}(m_1+m_2)\dot r^2+\frac{1}{2}m_1 r^2\dot\varphi^2$$

2p.

Megj.: $\dot r = v_r,\ r^2\dot\varphi^2 = v_\varphi$

6. $\frac{\partial L}{\partial r} = -m_2 g + m_1 r\dot\varphi^2$

   $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot r}\right) = (m_1+m_2)\ddot r$

   $\rightarrow (m_1+m_2)\ddot r = -m_2 g + m_1 r\dot\varphi^2$

   $\frac{\partial L}{\partial\varphi} = 0$

   $\frac{\partial L}{\partial\dot\varphi} = m_1 r^2\dot\varphi$

   $\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\varphi}\right) = 2m_1 r\dot r\dot\varphi + m_1 r^2\ddot\varphi$

   $\rightarrow 2m_1 r\dot r\dot\varphi + m_1 r^2\ddot\varphi = 0$

3 p.

Ezenkívül $\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\varphi}\right)=0$-ból következik, hogy $\frac{\partial L}{\partial\dot\varphi}=$ állandó.

$\rightarrow m_1 r^2\dot\varphi \equiv N =$ állandó

$\downarrow$

az 1-es test impulzusmomentuma

2p.