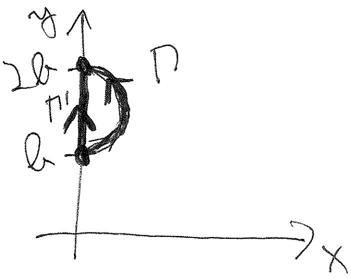
1.



$$\underline{F}(\underline{r}) = \frac{V_0}{b}\cdot\frac{3(\underline{b}\,\underline{r})\underline{r} - b^2\underline{b}}{r^3}, \quad \underline{b} = \begin{pmatrix} b \\ 0 \end{pmatrix}$$

$$W_\Gamma = ?$$

Mivel az erőtér konzervatív: $W_\Gamma = W_{\Gamma'}$

$$\Gamma': \; l = b, \quad \underline{r}(s) = \begin{pmatrix} x(s) \\ y(s) \end{pmatrix} = \begin{pmatrix} 0 \\ b+s \end{pmatrix}, \quad \underline{T}(s) = \frac{d\underline{r}(s)}{ds} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\underline{\underline{W_\Gamma}} = W_{\Gamma'} = \int_0^l \frac{V_0}{b}\cdot\frac{3\cdot\left((b,0)\begin{pmatrix} 0 \\ b+s \end{pmatrix}\right)\cdot(0, b+s) - b^2\cdot(b,0)}{(b+s)^3}\begin{pmatrix} 0 \\ 1 \end{pmatrix} ds \underline{\underline{= 0}}$$

$$(b,0)\begin{pmatrix} 0 \\ b+s \end{pmatrix} = 0 \qquad (b,0)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$$

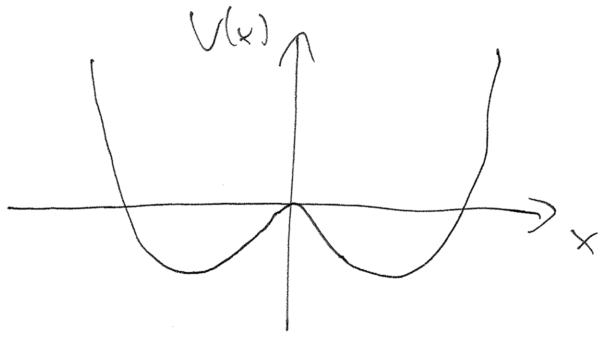
2.

$$m\ddot{x} = \gamma x - \delta x^3, \quad \gamma, \delta > 0$$

$$\rightarrow F(x) = \gamma x - \delta x^3$$

$$\rightarrow V(x) = -\int F(x)\,dx = -\frac{1}{2}\gamma x^2 + \frac{1}{4}\delta x^4 \cancel{+C}$$

legyen $C=0$



a) Ne lépje el az origót: $E \overset{!}{<} V(x=0) = 0$

$$E = \frac{1}{2} m \dot{x}^2 - \frac{1}{2}\gamma x^2 + \frac{1}{4}\delta x^4 = \frac{1}{2} m v_0^2 - \frac{1}{2}\gamma \cdot \frac{1}{4}\frac{\gamma}{\delta} + \frac{1}{4}\delta \cdot \frac{1}{16}\frac{\gamma^2}{\delta^2} =$$

az indításnál

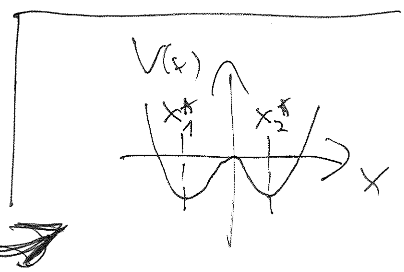
$$= \frac{1}{2} m v_0^2 - \frac{7}{64}\frac{\gamma^2}{\delta} \overset{!}{<} 0$$

$$v_0^2 \overset{!}{<} \frac{2}{m} \cdot \frac{7}{64}\frac{\gamma^2}{\delta} = \frac{7}{32} \cdot \frac{\gamma^2}{m\delta}, \quad \underline{\underline{|v_0| \overset{!}{<} \sqrt{\frac{7}{32}} \cdot \frac{\gamma}{\sqrt{m\delta}}}}$$

és $\underline{\underline{|v_0| \overset{!}{>} 0}}$ a b) feladathoz

b) $v_0 := 0$

- legnagyobb sebesség: ahol $V(x)$ minimális



$$V'(x) = -\gamma x + \delta x^3 \overset{!}{=} 0 \quad \to x^*_{1,2} = \mp\sqrt{\frac{\gamma}{\delta}}, \quad \to x^*_3 = 0$$

$$V''(x) = -\gamma + 3\delta x^2 \quad \to V''(x)\Big|_{x=x^*_{1,2}} = -\gamma + 3\delta \cdot \frac{\gamma}{\delta} = 2\gamma > 0$$

minimum

$$\to V''(x)\Big|_{x=x^*_3} = -\gamma < 0$$

maximum

Mivel tudjuk, hogy a tömegpont nem lépi el az origót, a tömegpont csak az origónak az indítás felöli (x_0 felöli) oldalán mozoghat, ez $x>0$-t jelent. A fenti szélsőértékek közül így egyedül $x_2^* = +\sqrt{\frac{\gamma}{\delta}}$-ba juthat el.

$$\dot{x}(x=x_2^*) = \pm\sqrt{\frac{2}{m}\left(E - V(x=x_2^*)\right)}$$

Mennyi is E?

$$\boxed{E = \frac{1}{2} m v_0^2 - \frac{7}{64}\frac{\gamma^2}{\delta} \underset{\substack{\uparrow \\ v_0=0}}{=} -\frac{7}{64}\frac{\gamma^2}{\delta}}$$

$$\rightarrow \underline{\underline{\dot{x}(x=x_2^*)}} = \pm\sqrt{\frac{2}{m}\cdot\left(-\frac{7}{64}\frac{\gamma^2}{\delta}+\frac{1}{2}\gamma\cdot\frac{\gamma}{\delta}-\frac{1}{4}\delta\frac{\gamma^2}{\delta^2}\right)} =$$

$$= \pm\sqrt{\frac{2}{m}\cdot\frac{16-7}{64}\cdot\frac{\gamma^2}{\delta}} = \underline{\underline{\pm\frac{3}{\sqrt{32}}\cdot\frac{\gamma}{\sqrt{m\cdot\delta}}}}$$

• Fordulópontok:

$$E = V(x_{fp}), \quad -\frac{7}{64}\frac{\gamma^2}{\delta} = -\frac{1}{2}\gamma x_{fp}^2 + \frac{1}{4}\delta x_{fp}^4$$

$$\rightarrow x_{fp}^2 = \frac{\frac{1}{2}\gamma \pm \sqrt{\frac{1}{4}\gamma^2 - 4\cdot\frac{1}{4}\delta\cdot\frac{7}{64}\frac{\gamma^2}{\delta}}}{2\cdot\frac{1}{4}\delta} =$$

$$= \left(\frac{1}{2}\gamma \pm \sqrt{\frac{16}{64}\gamma^2 - \frac{7}{64}\gamma^2}\right)\cdot\frac{2}{\delta} =$$

$$= \left(\frac{1}{2}\gamma \pm \frac{3}{8}\gamma\right)\cdot\frac{2}{\delta} =$$

$$= \left(1 \pm \frac{3}{4}\right)\cdot\frac{\gamma}{\delta}$$

A tömegpont csak az $x>0$ tartományban mozog, ezért csak a pozitív értékű fordulópontok kellenek:

$$x_{fp_1} = +\sqrt{\left(1-\frac{3}{4}\right)\frac{\gamma}{\delta}} = \frac{1}{2}\sqrt{\frac{\gamma}{\delta}}, \quad x_{fp_2} = +\sqrt{\left(1+\frac{3}{4}\right)\frac{\gamma}{\delta}} = \frac{\sqrt{7}}{2}\sqrt{\frac{\gamma}{\delta}}$$

(*) Nem meglepő: $x_0 = \frac{1}{2}\sqrt{\frac{\gamma}{\delta}}$-ban $\dot{x} = 0$ sebességgel indítottuk a tömegpontot, ezért x_0 fordulópont!

• Rajz: