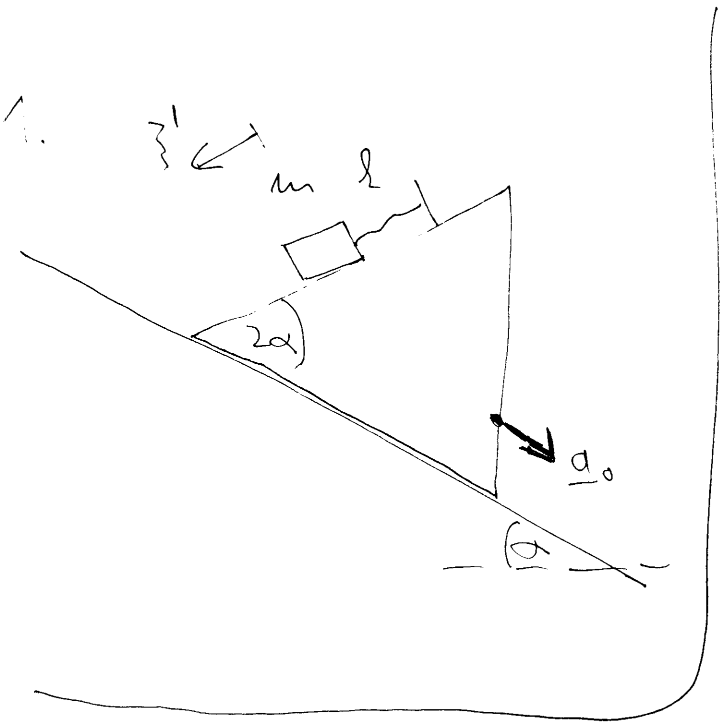
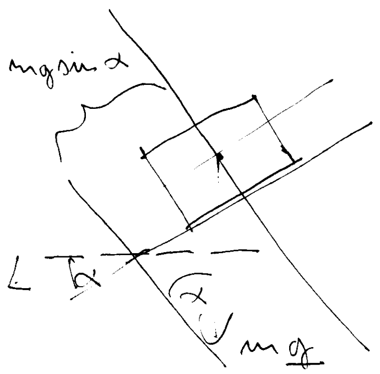
1.



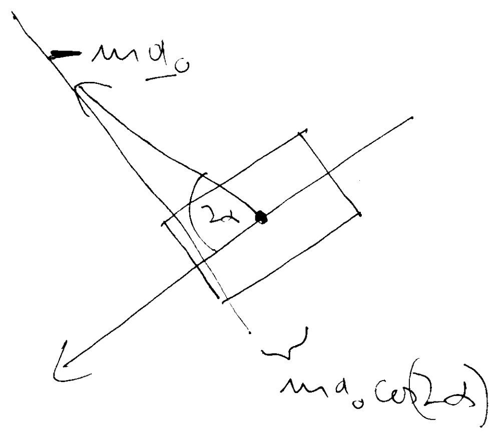
a) $m\underline{g} + \underline{N} + \underline{F}_{rugó} - m\underline{a}_0 = m\ddot{\underline{r}}'$



$(m\underline{g})_{\zeta'} = mg\sin\alpha$

$(\underline{N})_{\zeta'} = 0$

$(\underline{F}_{rugó})_{\zeta'} = -k\zeta$



$(-m\underline{a}_0)_{\zeta'} = m a_0 \cos(2\alpha)$

$\to \quad m\ddot{\zeta}' = mg\sin\alpha - k\zeta + m a_0\cos(2\alpha)$

b) $\ddot{\zeta}' \overset{!}{=} 0 \to \zeta^* = ?$

$\to \quad 0 = mg\sin\alpha - k\zeta^* + m a_0\cos(2\alpha)$

$$\zeta^* = \frac{m(g\sin\alpha + a_0\cos(2\alpha))}{k}$$

2.

$$x(t) = x_0 - \sqrt{x_0 a_0}\, t + \frac{1}{2} a_0 t^2$$

$\lim_{t \to \infty} x(t) = \infty$, vagyis elmegy a végtelenbe.

$$\dot{x}(t) = -\sqrt{x_0 a_0} + a_0 t$$

(1. mego.)

$$t(\dot{x}) = \frac{\dot{x} + \sqrt{x_0 a_0}}{a_0}$$

$$\rightarrow \quad x(\dot{x}) = \cancel{x_0} - \frac{\sqrt{x_0 a_0}}{a_0} \dot{x} - \cancel{\frac{x_0 a_0}{a_0}} + \frac{1}{2} a_0 \cdot \frac{1}{a_0^2} \cdot \left(\dot{x}^2 + 2\dot{x}\sqrt{x_0 a_0} + x_0 a_0\right) =$$

$$= -\cancel{\frac{\sqrt{x_0 a_0}}{a_0}\dot{x}} + \frac{1}{2a_0}\dot{x}^2 + \cancel{\dot{x}\frac{\sqrt{x_0 a_0}}{a_0}} + \frac{1}{2}x_0 =$$

$$= \frac{1}{2a_0}\dot{x}^2 + \frac{1}{2}x_0$$

$$\frac{1}{2a_0}\dot{x}^2 = x - \frac{1}{2}x_0$$

felvő parabola

Fordulópont:

$$\dot{x} = 0 \quad \rightarrow \quad 0 = x_{fp} - \frac{1}{2}x_0 \;; \quad x_{fp} = \frac{1}{2}x_0$$

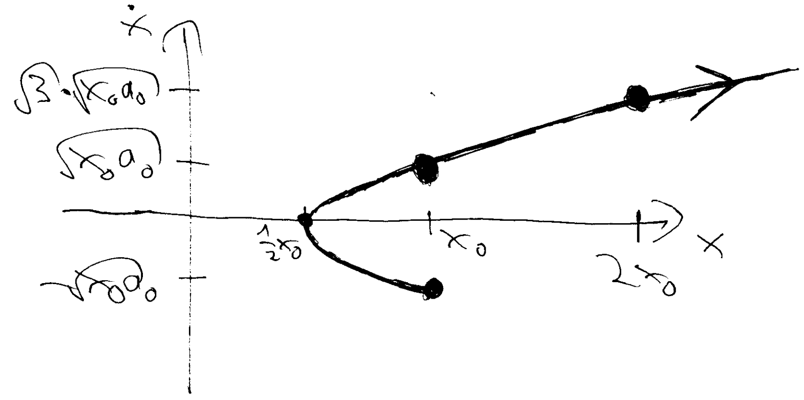
Sebesség:

$$x = 2x_0 \rightarrow \dot{x}(x = 2x_0): \quad \frac{1}{2a_0}\dot{x}(x=2x_0)^2 = 2x_0 - \frac{1}{2}x_0$$

$$\dot{x}(x=2x_0)^2 = 2a_0 \cdot \frac{3}{2}x_0 = 3a_0 x_0 \;, \quad \dot{x}(x=2x_0) = \pm\sqrt{3a_0 x_0}$$

DE a kezdeti feltétel szerint csak jobbra haladva megy itt át! $\Rightarrow \dot{x} > 0$

Rajz: $t=0: \; x = x_0, \; \dot{x} = -\sqrt{x_0 a_0}$



2. mego.

$$\ddot{x}(t) = a_0$$

$$m\ddot{x} = -V'(x) \Rightarrow V(x) = -m a_0 x$$

(a konstans választható 0-nak)

$$E = \frac{1}{2} m \dot{x}^2 + V(x) = \frac{1}{2} m x_0 a_0 - m a_0 x_0 = -\frac{1}{2} m a_0 x_0$$

↑

$t=0: \; x = x_0, \; \dot{x} = \sqrt{x_0 a_0}$

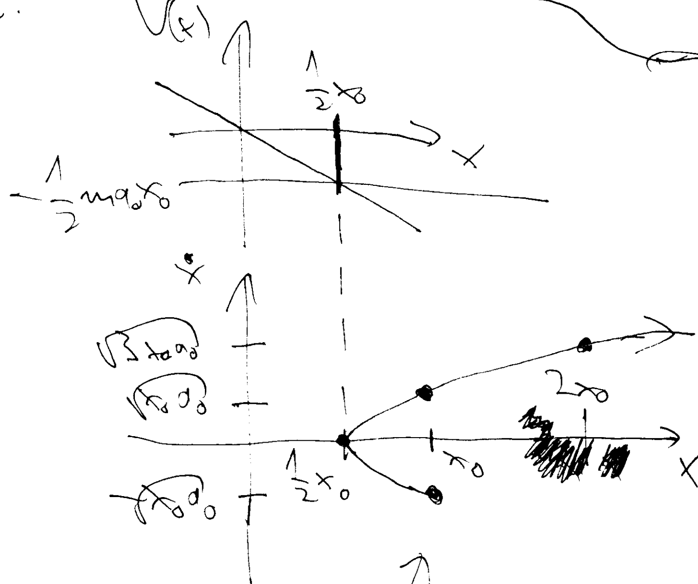
Fordulópont: $E = V(x_{fp}) \rightarrow -\frac{1}{2} m a_0 x_0 = -m a_0 x_{fp}, \; x_{fp} = \frac{1}{2} x_0$

Sebesség: $\dot{x}(x=2x_0) = \pm\sqrt{\frac{2}{m}\left(E - V(x=x_0)\right)} = \pm\sqrt{\frac{2}{m}\left(-\frac{1}{2} m a_0 x_0 + m a_0 2x_0\right)} =$

$= \pm\sqrt{3 a_0 x_0}$

DE a kezdeti feltétel miatt itt $\dot{x} > 0, \; \dot{x}(x=2x_0) = \sqrt{3 a_0 x_0}$

Rajz:



$t=0: \; x = x_0, \; \dot{x} = -\sqrt{x_0 a_0}$

Egyenlet:

$$-\frac{1}{2} m a_0 x_0 = \frac{1}{2} m \dot{x}^2 - m a_0 x$$

„fekvő parabola"

3.

$$v_{f\,r} = v_0(2+\cos\varphi)$$

$$v_{f\varphi} = \frac{v_0}{R_2} r$$

$$t=0 \;:\; r = R_1,\ \varphi = \varphi_0$$

$$v_{rel\,r} = -u_0(2+\cos\varphi)$$

$$v_{rel\,\varphi} = -\frac{(v_0-u_0)^2}{R_2} \cdot \frac{r(2+\cos\varphi)}{v_r}$$

$$r(\varphi = \varphi_0 + \theta) = R_2$$

$$x_0 = R_1 \cos\varphi_0 = ?$$

$$y_0 = R_1 \sin\varphi_0 = ?$$

$$v_r = v_{fr} + v_{rel\,r} = (v_0 - u_0)(2+\cos\varphi)$$

$$v_\varphi = v_{f\varphi} + v_{rel\,\varphi} = \frac{v_0}{R_2} r - \frac{(v_0-u_0)^{\cancel{2}}}{R_2} \cdot \frac{r\cancel{(2+\cos\varphi)}}{\cancel{(v_0-u_0)}\cdot\cancel{(2+\cos\varphi)}} =$$

$$= \frac{v_0-(v_0-u_0)}{R_2} \cdot r = \frac{u_0}{R_2} \cdot r$$

$$\varphi(t)=\varphi(t_0)+\int_{t_0}^{t}\frac{v_\varphi(t')}{r(t')}\,dt'=\varphi_0+\int_0^t\frac{\frac{u_0}{R_2}\cdot\cancel{r(t')}}{\cancel{r(t')}}\,dt'=$$

$$=\varphi_0+\frac{u_0}{R_2}\cdot t \quad (*)$$

$$r(t)=r(t_0)+\int_{t_0}^{t}v_r(t')\,dt'=R_1+\int_0^t(v_0-u_0)\left(2+\cos\varphi(t')\right)dt'=$$

$$=R_1+(v_0-u_0)\int_0^t\left(2+\cos\left(\varphi_0+\frac{u_0}{R_2}\cdot t'\right)\right)dt'=$$

$$=R_1+2(v_0-u_0)t+(v_0-u_0)\frac{R_2}{u_0}\left(\sin\left(\varphi_0+\frac{u_0}{R_2}t\right)-\sin\varphi_0\right)$$

$$(*)\rightarrow\ t(\varphi)=(\varphi-\varphi_0)\cdot\frac{R_2}{u_0}$$

$$\rightarrow\ r(\varphi)\equiv r(t(\varphi))=R_1+2(v_0-u_0)\cdot\frac{R_2}{u_0}(\varphi-\varphi_0)+(v_0-u_0)\cdot\frac{R_2}{u_0}(\sin\varphi-\sin\varphi_0)$$

$$r(\varphi=\varphi_0+\phi)=R_2$$

$$\rightarrow\ R_2=R_1+2(v_0-u_0)\cdot\frac{R_2}{u_0}\cdot\phi+(v_0-u_0)\cdot\frac{R_2}{u_0}\left(\sin(\varphi_0+\phi)-\sin\varphi_0\right)$$

$\rightarrow$ „$\varphi_0=?$"

Nem fejezhető ki,
de nem is szükséges,
mert valójában:

$x_0=R_1\cos\varphi_0=?$

$y_0=R_1\sin\varphi_0=?$

$$R_2 = R_1 + 2(v_0-u_0)\frac{R_2}{u_0}\phi + (v_0-u_0)\frac{R_2}{u_0}\cdot\frac{1}{R_1}\cdot\Big(\underbrace{R_1\sin\varphi_0}_{y_0}\cos\phi - \overbrace{R_1\cos\varphi_0}^{x_0}\sin\phi - \underbrace{R_1\sin\varphi_0}_{y_0}\Big)$$

Másrészt: $x_0^2+y_0^2=R_1^2$

Ha sikerül beírni ide az egyenlet a változókkal kifejezve, akkor egy másodfokú egyenletünk lesz!

Eddig elég eljutni!!!

$$\overbrace{\frac{R_2-R_1+2(v_0-u_0)\frac{R_2}{u_0}\phi}{(v_0-u_0)\frac{R_2}{u_0}\cdot\frac{1}{R_1}}}^{=:C} = y_0(\cos\phi-1) - x_0\sin\phi$$

$$y_0 = x_0\frac{\sin\phi}{\cos\phi-1} + \frac{C}{\cos\phi-1}$$

$$\rightarrow \quad x_0^2 + x_0^2\left(\frac{\sin\phi}{\cos\phi-1}\right)^2 + 2x_0\frac{\sin\phi}{(\cos\phi-1)^2}\cdot C + \frac{C^2}{(\cos\phi-1)^2} = R_1^2$$

$$\underbrace{\left(1+\left(\frac{\sin\phi}{\cos\phi-1}\right)^2\right)}_{=:\tilde a}x_0^2 + \underbrace{\frac{2\sin\phi\cdot C}{(\cos\phi-1)^2}}_{=:\tilde b}x_0 + \underbrace{\frac{C^2}{(\cos\phi-1)^2}-R_1^2}_{=:\tilde c} = 0$$

$$\rightarrow \quad x_0 = \frac{-\tilde b \pm\sqrt{\tilde b^2 - 4\tilde a\tilde c}}{2\tilde a}$$
, és y_0 innen már adódik.

Mivel $\cos\varphi$ nem kölcsönösen egyértelmű, valóban lehet 2 megoldás.

Érdemes észrevenni:

$$C = \left[\left(1-\frac{R_1}{R_2}\right)\frac{u_0}{v_0-u_0}+2\phi\right]\cdot R_1$$

Tehát a mértékegységek rendben vannak!

Vegyük észre, hogy a mozgást $v_r=(v_0-u_0)(2+\cos\varphi)$ és $\dot\varphi=\frac{u_0}{R_2}$ írja le.