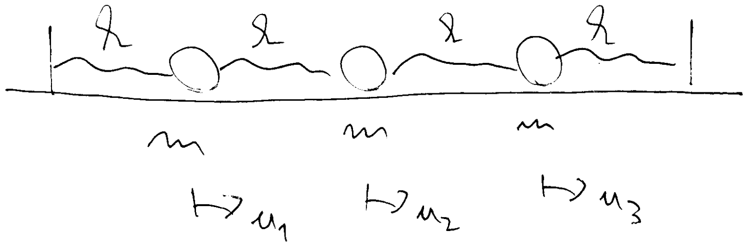
1.



$$\ddot{u}_1 = -\frac{2k}{m}u_1 + \frac{k}{m}u_2$$
$$\ddot{u}_2 = \frac{k}{m}u_1 - \frac{2k}{m}u_2 + \frac{k}{m}u_3$$
$$\ddot{u}_3 = \frac{k}{m}u_2 - \frac{2k}{m}u_3$$

Ansatz:

$$u_1(t) = A_1 e^{i\omega t}$$
$$u_2(t) = A_2 e^{i\omega t}$$
$$u_3(t) = A_3 e^{i\omega t}$$
$$\lambda := -\omega^2$$

Sajátért. egy.:

$$\begin{vmatrix} -\frac{2k}{m}-\lambda & \frac{k}{m} & 0 \\ \frac{k}{m} & -\frac{2k}{m}-\lambda & \frac{k}{m} \\ 0 & \frac{k}{m} & -\frac{2k}{m}-\lambda \end{vmatrix} = \left(-\frac{2k}{m}-\lambda\right)^3 - \left(-\frac{2k}{m}-\lambda\right)\cdot\frac{k^2}{m^2} - \left(-\frac{2k}{m}-\lambda\right)\frac{k^2}{m^2} = 0$$

$$\Rightarrow -\frac{2k}{m} - \lambda_I = 0$$

$$\lambda_I = -\frac{2k}{m} \Rightarrow \underline{\omega_{I,+} = \sqrt{2\frac{k}{m}}}$$

$$\left(-\frac{2k}{m} - \lambda_{II,III}\right)^2 - 2\cdot\frac{k^2}{m^2} = 0$$

$$-\frac{2k}{m} - \lambda_{II,III} = \mp\sqrt{2}\frac{k}{m}$$

$$\lambda_{II,III} = -2\frac{k}{m} \pm \sqrt{2}\frac{k}{m}$$

$$\Rightarrow \omega_{II,+} = \sqrt{(2-\sqrt{2})\frac{k}{m}}$$
$$\underline{\omega_{III,+} = \sqrt{(2+\sqrt{2})\frac{k}{m}}}$$

2.

$$V(r) = -\frac{1}{2}\gamma(r-d)^2 \rightarrow V_{eff}(r) = \frac{N^2}{2mr^2} - \frac{1}{2}\gamma(r-d)^2, \text{ de } N = ?$$

Körpálya: $r(t) = r^*$, $v_r \equiv \dot{r} = 0 \Rightarrow |\underline{v}| = |v_\varphi| =: w$ ← jelölés

Mivel a kérdés a sebességre vonatkozik, érdemes ezzel kifejezni N-et:

$$|N| = mr|v_\varphi| = mr^*w \quad \rightarrow \quad V_{eff}(r) = \frac{m^2 r^{*2} w^2}{2mr^2} - \frac{1}{2}\gamma(r-d)^2$$

a körpályán

Most vizsgáljuk meg a körpálya feltételét:

$$V'_{eff}(r) = -\frac{m^2 r^{*2} w^2}{m r^3} - \gamma(r-d) = -\frac{m r^{*2} w^2}{r^3} - \gamma r + \gamma d$$

$$V'_{eff}(r)\Big|_{r=r^*} = -\frac{m \cancel{r^{*2}} w^2}{r^{*\cancel{3}}} - \gamma r^* + \gamma d = 0$$

$$-\gamma r^{*2} + \gamma d r^* - m w^2 = 0$$

$$r^*_{1,2} = \frac{-\gamma d \pm \sqrt{\gamma^2 d^2 - 4\gamma m w^2}}{-2\gamma}$$

Akkor van megoldása, ha:

$$\gamma^2 d^2 - 4\gamma m w^2 \geq 0$$

$$4 m w^2 \leq \gamma d^2$$

$$w \leq \frac{d}{2}\sqrt{\frac{\gamma}{m}}$$

A legnagyobb sebesség tehát:

$$\underline{\underline{|\underline{v}| = w = \frac{d}{2}\sqrt{\frac{\gamma}{m}}}}$$

A vonatkozó körpálya sugara:

$$r^* = \frac{-\gamma d \pm 0}{-2\gamma} = \frac{d}{2}$$

Szögsebesség:

$$|\Omega^*| = \frac{w}{r^*} = \sqrt{\frac{\gamma}{m}}$$

Stabilitás, kis rezgések:

$$V''_{eff}(r) = 3\,\frac{m r^{*2} w^2}{r^4} - \gamma$$

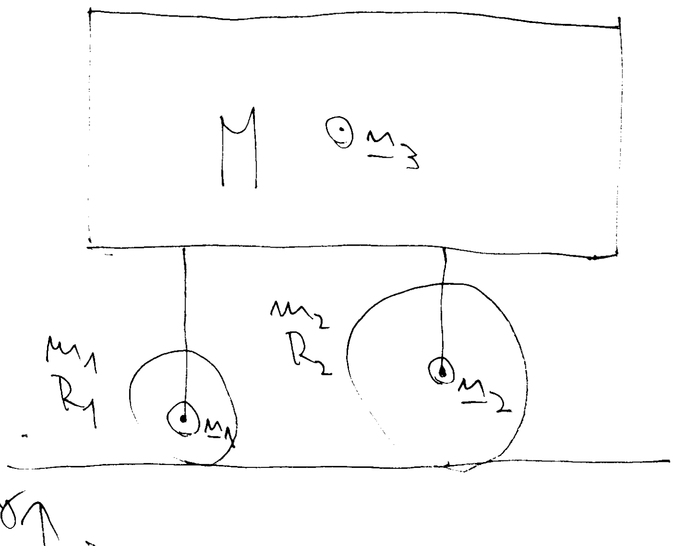
$$V''_{eff}(r)\Big|_{r=r^*} = 3\,\frac{m w^2}{r^{*2}} - \gamma = 3 \cdot \frac{m \cdot \frac{d^2}{4} \cdot \frac{\gamma}{m}}{\frac{d^2}{4}} - \gamma = 3\gamma - \gamma = 2\gamma > 0$$

Igen, <u>stabil</u>.

$$\omega_{r^*} = \sqrt{\frac{V''_{eff}(r)\Big|_{r=r^*}}{m}} = \sqrt{2\frac{\gamma}{m}}$$

$$\omega_{r^*}/|\Omega^*| = \sqrt{2} \notin \mathbb{Q} \Rightarrow$$ <u>Nem, nem záródik.</u>

3.



1.) $x_1, y_1, \varphi_1, x_2, y_2, \varphi_2, x_3, y_3, \varphi_3$

2.)
- y_1, y_2 és y_3 állandó
- $\varphi_3 =$ állandó
- $x_1 - x_3 =$ állandó, $x_2 - x_3 =$ állandó
- tiszta gördülés, <u>mindkét</u> hengernél

3.) x_3

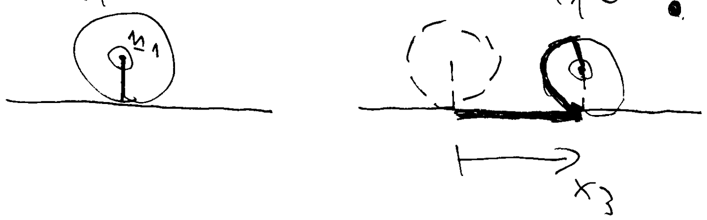
4.) $x_1 = x_3 + c_1$

$y_1 = d_1$

$\varphi_1 = ?$

Legyen $\varphi_1 = 0$ akkor, amikor $x_3 = 0$. Ehhez a választáshoz jogunk van.

$\varphi_1 = 0$ $\quad\quad$ $\varphi_1 < 0$! (Hagyományosan $\oplus$)



$-R_1\varphi_1 = x_3$

$\varphi_1 = -\frac{x_3}{R_1}$

$x_2 = x_3 + c_2$

$y_2 = d_2$

$\varphi_2 = -\frac{x_3}{R_2}$ (hasonlóan φ_1-hez)

$x_3 = x_3$

$y_3 = d_3$

$\varphi_3 = \phi =$ állandó

$\dot{x}_1 = \dot{x}_3$

$\dot{y}_1 = 0$

$\dot{\varphi}_1 = -\frac{\dot{x}_3}{R_1}$

$\dot{x}_2 = \dot{x}_3$

$\dot{y}_2 = 0$

$\dot{\varphi}_2 = -\frac{\dot{x}_3}{R_2}$

$\dot{x}_3 = \dot{x}_3$

$\dot{y}_3 = 0$

$\dot{\varphi}_3 = 0$

5.) $L = T - V$

$$T = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}\Theta_{m_1}\dot{\varphi}_1^2 + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2) + \frac{1}{2}\Theta_{m_2}\dot{\varphi}_2^2 +$$
$$+ \frac{1}{2}M(\dot{x}_3^2 + \dot{y}_3^2) + \frac{1}{2}\Theta_{m_3}\dot{\varphi}_3^2 = \frac{1}{2}m_1\dot{x}_3^2 + \frac{1}{2}\cdot\frac{1}{2}m_1R_1^2\cdot\frac{\dot{x}_3^2}{R_1^2} +$$
$$+ \frac{1}{2}m_2\dot{x}_3^2 + \frac{1}{2}\cdot\frac{1}{2}m_2R_2^2\cdot\frac{\dot{x}_3^2}{R_2^2} + \frac{1}{2}M\dot{x}_3^2 + 0 =$$
$$= \left(\frac{3}{4}m_1 + \frac{3}{4}m_2 + \frac{1}{2}M\right)\dot{x}_3^2$$

$V = m_1 g y_1 + m_2 g y_2 + M g y_3 =$ konstans

6.) $\frac{\partial L}{\partial x_3} = 0$

$$p_{x_3} \equiv \frac{\partial L}{\partial \dot{x}_3} = \left(\frac{3}{2}m_1 + \frac{3}{2}m_2 + M\right)\dot{x}_3$$

A kezdeti feltételek határozzák meg.

$$\rightarrow \underline{\underline{\left(\frac{3}{2}m_1 + \frac{3}{2}m_2 + M\right)\dot{x}_3 = C = \text{állandó}}}$$

Vagy másként: $\left(\frac{3}{2}m_1 + \frac{3}{2}m_2 + M\right)\ddot{x}_3 = 0$