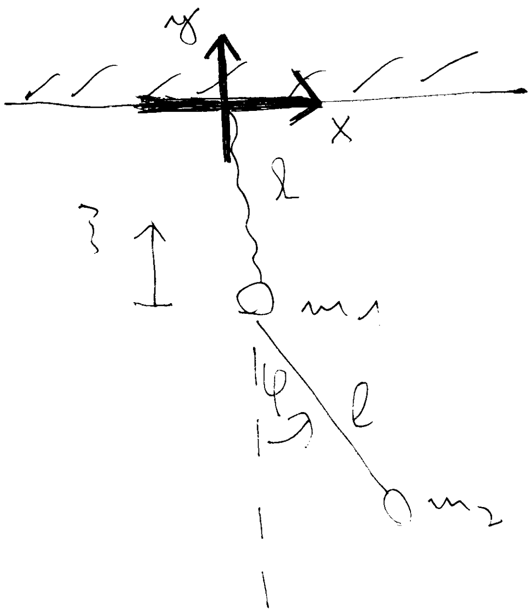
ZH2Pót/1

1.



1.) x_1, y_1, x_2, y_2

2.) • $x_1 = 0$

• ingoriód

3.) ξ, φ

4.) $x_1 = 0$

$y_1 = c_1 + \xi$

$x_2 = l \sin\varphi$

$y_2 = c_1 + \xi - l\cos\varphi$

$\dot{x}_1 = 0$

$\dot{y}_1 = \dot{\xi}$

$\dot{x}_2 = l\cos\varphi\,\dot{\varphi}$

$\dot{y}_2 = \dot{\xi} + l\sin\varphi\,\dot{\varphi}$

5.) $L = T - V$

$$T = \frac{1}{2}m_1(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m_2(\dot{x}_2^2 + \dot{y}_2^2) =$$

$$= \frac{1}{2}m_1\dot{\xi}^2 + \frac{1}{2}m_2\big(l^2\cos^2\varphi\,\dot{\varphi}^2 +$$

$$+ \dot{\xi}^2 + 2\dot{\xi}\,l\sin\varphi\,\dot{\varphi} + l^2\sin^2\varphi\,\dot{\varphi}^2\big) =$$

$$= \frac{1}{2}(m_1 + m_2)\dot{\xi}^2 + m_2 l\,\dot{\xi}\sin\varphi\,\dot{\varphi} + \frac{1}{2}m_2 l^2\dot{\varphi}^2$$

$$V = m_1 g y_1 + m_2 g y_2 + \frac{1}{2}k\xi^2 = (m_1 + m_2)g\xi - m_2 g l\cos\varphi + \frac{1}{2}k\xi^2 + \text{konstans}$$

6.) $$\frac{\partial L}{\partial \xi} = -(m_1 + m_2)g - k\xi$$

$$\frac{\partial L}{\partial \dot{\xi}} = (m_1 + m_2)\dot{\xi} + m_2 l\sin\varphi\,\dot{\varphi}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\xi}}\right) = (m_1 + m_2)\ddot{\xi} + m_2 l\cos\varphi\,\dot{\varphi}^2 + m_2 l\sin\varphi\,\ddot{\varphi}$$

$$\rightarrow (m_1+m_2)\ddot{\xi} + m_2 l \cos\varphi\, \dot{\varphi}^2 + m_2 l \sin\varphi\, \ddot{\varphi} =$$
$$= -(m_1+m_2)g - k\xi$$

$$\frac{\partial L}{\partial \varphi} = m_2 l \dot{\xi} \cos\varphi\, \dot{\varphi} + m_2 g l \cdot (-\sin\varphi)$$

$$\frac{\partial L}{\partial \dot{\varphi}} = m_2 l \dot{\xi} \sin\varphi + m_2 l^2 \dot{\varphi}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) = m_2 l \ddot{\xi} \sin\varphi + m_2 l \dot{\xi} \cos\varphi\, \dot{\varphi} + m_2 l^2 \ddot{\varphi}$$

$$\rightarrow m_2 \cancel{l} \ddot{\xi} \sin\varphi + m_2 \cancel{l} \dot{\xi} \cos\varphi\, \dot{\varphi} + m_2 l^{\cancel{2}} \ddot{\varphi} =$$
$$= m_2 \cancel{l} \dot{\xi} \cos\varphi\, \dot{\varphi} - m_2 g \cancel{l} \sin\varphi$$

2.

$V(r)=\frac{1}{2}k r^2$

A tömegpont teljes sebessége az energiamegmaradásból kapható:

$$E=\frac{1}{2}m|\underline{v}|^2+V(r)$$

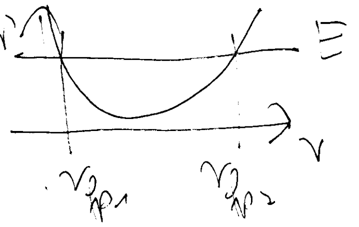
$(|\underline{v}|\equiv v)$

$$\to \quad v=\sqrt{\frac{2}{m}\left(E-V(r)\right)}$$

Mivel $V(r)$ szig. mon. növekszik r függvényében, $\max\{v\}$ ott lesz, ahol r minimális, tehát ahol a tömegpont a legközelebb kerül az origóhoz. Ez az effektív potenciálban leírt mozgás fordulópontja.

$$V_{eff}(r)=\frac{N^2}{2mr^2}+V(r)=\frac{3}{4}E^2\frac{\cancel{m}}{k}\cdot\frac{1}{2\cancel{m}}\cdot\frac{1}{r^2}+\frac{1}{2}kr^2=$$

$$=\frac{3}{8}\frac{E^2}{k}\frac{1}{r^2}+\frac{1}{2}kr^2$$



$$V_{eff}(r=r_{fp})=E$$

$$\Rightarrow \quad \frac{3}{8}\frac{E^2}{k}\frac{1}{r_{fp}^2}+\frac{1}{2}kr_{fp}^2=E$$

$$\frac{1}{2}kr_{fp}^4-Er_{fp}^2+\frac{3}{8}\frac{E^2}{k}=0$$

$$\rightarrow\; r^2_{fp_{2,1}} = \frac{E \pm \sqrt{E^2 - 4\cdot\frac{1}{2}\ell\cdot\frac{3}{8}\frac{E^2}{\ell}}}{\ell} =$$

$$= \frac{E \pm \sqrt{E^2 - \frac{3}{4}E^2}}{\ell} = \frac{E \pm \frac{1}{2}E}{\ell}$$

Mi a kisebb értékű fordulópontot keressük:

$$r^2_{fp1} = \frac{E - \frac{1}{2}E}{\ell} = \frac{1}{2}\cdot\frac{E}{\ell}, \quad \boxed{r_{fp1} = \sqrt{\frac{1}{2}\cdot\frac{E}{\ell}}}$$

$$\rightarrow\; \max\{v\} = \sqrt{\frac{2}{m}\left(E - V(r = r_{fp1})\right)} =$$

$$= \sqrt{\frac{2}{m}\left(E - \frac{1}{2}\cancel{\ell}\cdot\frac{1}{2}\cdot\frac{E}{\cancel{\ell}}\right)} = \sqrt{\frac{2}{m}\cdot\frac{3}{4}E} = \underline{\underline{\sqrt{\frac{3}{2}}\cdot\sqrt{\frac{E}{m}}}}$$

Szögsebesség:

Mivel fordulópontban vagyunk, $v_r = 0$, így $|v_\varphi| = v$.

v_φ előjele N előjelével egyezik meg, ami negatív, mivel $E > 0$ (a potenciálalak miatt).

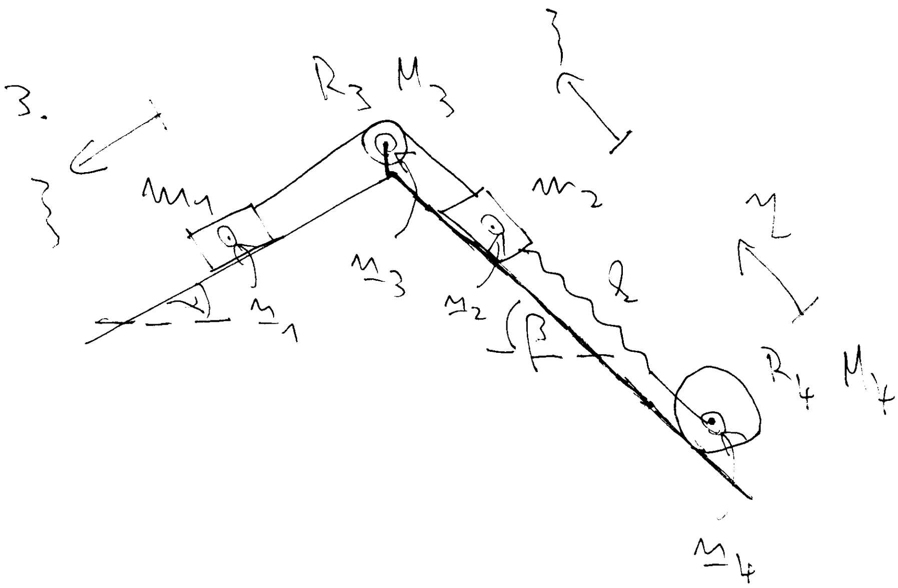
$$\rightarrow\; v_\varphi \overset{\text{itt}}{=} -v = -\sqrt{\frac{3}{2}}\cdot\sqrt{\frac{E}{m}}$$

$$\dot\varphi = \frac{v_\varphi}{r} \underset{\text{itt}}{=} -\frac{\sqrt{3}}{\sqrt{2}}\cdot\sqrt{\frac{E}{m}}\cdot\frac{1}{r_{fp1}} = -\frac{\sqrt{3}}{\sqrt{2}}\cdot\sqrt{\frac{E}{m}}\cdot\sqrt{2}\cdot\sqrt{\frac{\ell}{E}} = \underline{\underline{-\sqrt{3}\sqrt{\frac{\ell}{m}}}}$$

Más úton:

$$\dot\varphi = \frac{N}{mr^2} \underset{\text{itt}}{=} \frac{N}{m r^2_{fp1}} = -\frac{\sqrt{3}}{2}\cancel{E}\sqrt{\frac{m}{\ell}}\cdot\frac{1}{m}\cdot 2\frac{\ell}{\cancel{E}} = \underline{\underline{-\sqrt{3}\sqrt{\frac{\ell}{m}}}}$$

3.



1.) x_1, y_1, φ_1,

x_2, y_2, φ_2,

x_3, y_3, φ_3,

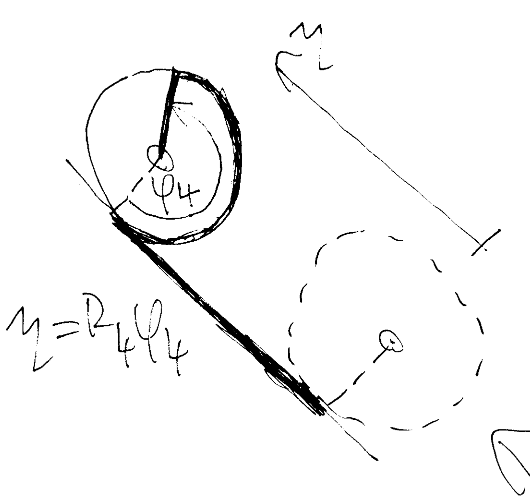
x_4, y_4, φ_4

2.)
- φ_1 = állandó
- φ_2 = állandó
- lejtő (a hasábokra)
- a csiga ~~tengelye~~ rögzített
- lejtő (a hengerre)
- tiszta gördülés (a hengerre)
- a fonál geometriai kapcsolata a csigával
- a fonál geometriai kapcsolata a hasábokkal

3.) ξ, η

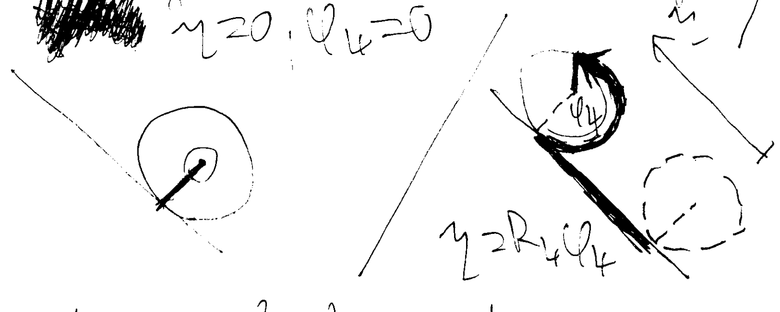
Legyen $\xi = \eta$ akkor, amikor a rugó nyújtatlan (definiálhatom így ezt a két mennyiséget).

Legyen továbbá $\varphi_3 = 0$, amikor $\xi = 0$, és legyen $\varphi_4 = 0$, amikor $\eta = 0$ (definiálhatom így a referenciaszituációt!).



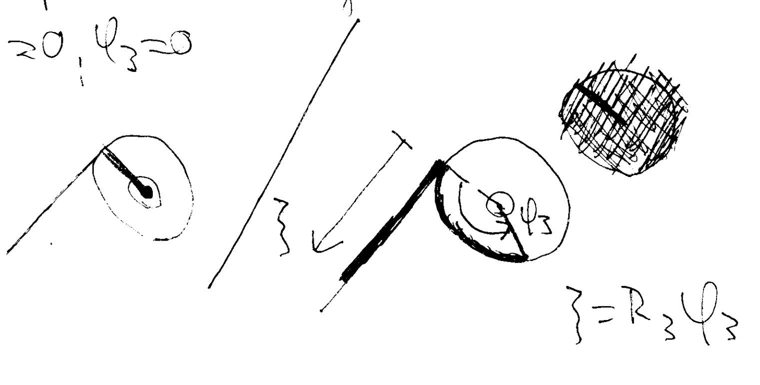
A tiszta gördülés geometriai viszonyai:

$\eta = 0$, $\varphi_4 = 0$



Hasonlóan: a fonál geometriai kapcsolata a csigával:

$\xi = 0$, $\varphi_3 = 0$



4.)

$x_1 = c_1 - \xi \cos\alpha$ $\qquad$ $\dot{x}_1 = -\dot{\xi}\cos\alpha$

$y_1 = d_1 - \xi \sin\alpha$ $\qquad$ $\dot{y}_1 = -\dot{\xi}\sin\alpha$

$\varphi_1 = \phi_1 = \text{állandó}$ $\qquad$ $\dot{\varphi}_1 = 0$

$x_2 = c_2 - \xi \cos\beta$ $\qquad$ $\dot{x}_2 = -\dot{\xi}\cos\beta$

$y_2 = d_2 + \xi \sin\beta$ $\qquad$ $\dot{y}_2 = +\dot{\xi}\sin\beta$

$\varphi_2 = \phi_2 = \text{állandó}$ $\qquad$ $\dot{\varphi}_2 = 0$

$x_3 = c_3 = \text{állandó}$ $\qquad$ $\dot{x}_3 = 0$

$y_3 = d_3 = \text{állandó}$ $\qquad$ $\dot{y}_3 = 0$

$\varphi_3 = \xi / R_3$ $\qquad$ $\dot{\varphi}_3 = \dot{\xi}/R_3$

$x_4 = c_4 - \eta\cos\beta$ $\qquad$ $\dot{x}_4 = -\dot{\eta}\cos\beta$

$y_4 = d_4 + \eta\sin\beta$ $\qquad$ $\dot{y}_4 = \dot{\eta}\sin\beta$

$\varphi_4 = \eta / R_4$ $\qquad$ $\dot{\varphi}_4 = \dot{\eta}/R_4$

5.) $L = T - V$

$$T = \frac{1}{2} m_1 \dot{\xi}^2 + \frac{1}{2} m_2 \dot{\xi}^2 + \frac{1}{2} M_3 \cdot 0^2 + \frac{1}{2} \underbrace{\Theta_{M_3}}_{\Theta_{M_3} = \frac{1}{2} M_3 R_3^2} \left(\frac{\dot{\xi}}{R_3}\right)^2 + \frac{1}{2} M_4 \dot{\eta}^2 + \frac{1}{2} \underbrace{\Theta_{M_4}}_{\Theta_{M_4} = \frac{1}{2} M_4 R_4^2} \left(\frac{\dot{\eta}}{R_4}\right)^2 =$$

$$= \left(\frac{1}{2} m_1 + \frac{1}{2} m_2 + \frac{1}{4} M_3\right)\dot{\xi}^2 + \frac{3}{4} M_4 \dot{\eta}^2$$

$$V = m_1 g y_1 + m_2 g y_2 + M_3 g y_3 + M_4 g y_4 + \frac{1}{2} k(\xi - \eta)^2 =$$

$$= -m_1 g \sin\alpha \cdot \xi + m_2 g \sin\beta \cdot \xi + M_4 g \sin\beta \cdot \eta + \frac{1}{2} k(\xi - \eta)^2 + \text{konstans}$$

6.) $\frac{\partial L}{\partial \xi} = m_1 g \sin\alpha - m_2 g \sin\beta - k(\xi - \eta)\cdot 1$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\xi}}\right) = \left(m_1 + m_2 + \frac{1}{2}M_3\right)\ddot{\xi}$

$\rightarrow \quad \left(m_1 + m_2 + \frac{1}{2}M_3\right)\ddot{\xi} = m_1 g \sin\alpha - m_2 g \sin\beta - k(\xi - \eta)$

$\frac{\partial L}{\partial \eta} = -M_4 g \sin\beta - k(\xi - \eta)\cdot(-1)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\eta}}\right) = \frac{3}{2}M_4 \ddot{\eta}$

$\rightarrow \quad \frac{3}{2}M_4 \ddot{\eta} = -M_4 g \sin\beta + k(\xi - \eta)$