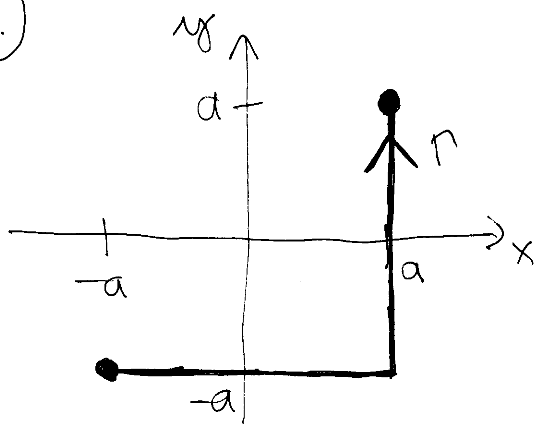
ZH1H/1

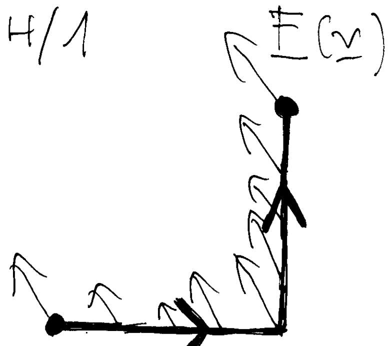
1.



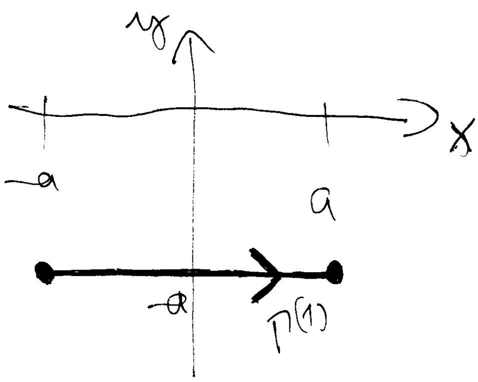
$$\underline{F}(\underline{r}) = Cr^2\underline{e}_-$$

$$C>0$$

$$W_\Gamma = ?$$



Az erőtérben és a tekintett görbében jelenlévő szimmetria miatt a munkavégzést 0-nak várjuk. (Részletes indoklással ez is elfogadható eredményként.)



$\Gamma^{(1)}$: $l=2a$, $x(s)=-a+s$, $y(s)=-a$,

$$\underline{T}(s)=\frac{d}{ds}\begin{pmatrix}-a+s\\-a\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$$

$$\int_{\Gamma^{(1)}}\underline{F}(\underline{r})\,d\underline{r}=\int_{\Gamma^{(1)}}Cr^2\underline{e}_-\,d\underline{r}=C\int_0^{2a}(x^2+y^2)\,\underline{e}_-\cdot\underline{T}\,ds=$$

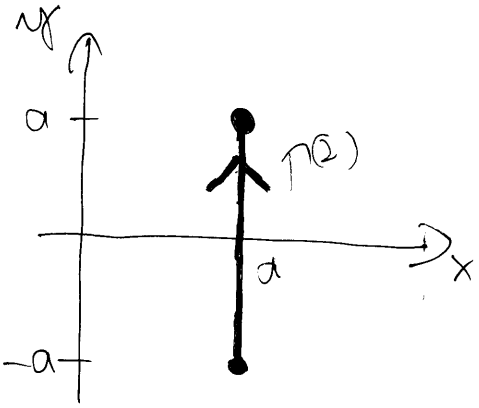
$$=\frac{C}{\sqrt{2}}\int_0^{2a}\left((-a+s)^2+(-a)^2\right)\cdot\underbrace{(-1,1)\cdot\begin{pmatrix}1\\0\end{pmatrix}}_{=-1}ds=$$

$$=-\frac{C}{\sqrt{2}}\int_0^{2a}\left(2a^2-2as+s^2\right)ds=-\frac{C}{\sqrt{2}}\left[2a^2s-2a\frac{s^2}{2}+\frac{s^3}{3}\right]_0^{2a}=$$

$$=-\frac{C}{\sqrt{2}}\left(2a^2\cdot 2a-2a\cdot\frac{4a^2}{2}+\frac{8a^3}{3}\right)=-\frac{C}{\sqrt{2}}a^3\cdot\left(4-4+\frac{8}{3}\right)=$$

$$=-\frac{8}{3\sqrt{2}}Ca^3$$

ZH1H/2



$\Gamma^{(2)}: \ \ell = 2a, \ x(s) = a, \ y(s) = -a+s,$

$$\underline{T}(s) = \frac{d}{ds}\begin{pmatrix} a \\ -a+s \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\int_{\Gamma^{(2)}} \underline{E}(\underline{r})\, d\underline{r} = \frac{C}{\sqrt{2}} \int_0^{2a} (a^2 + (-a+s)^2) \overbrace{(-1,1)\cdot\begin{pmatrix} 0 \\ 1 \end{pmatrix}}^{=1} ds =$$

$$= \frac{C}{\sqrt{2}} \int_0^{2a} (2a^2 - 2as + s^2)\, ds = -\int_{\Gamma^{(1)}} \underline{E}(\underline{r})\, d\underline{r} = \frac{8}{3\sqrt{2}} C a^3$$

$$W_\Gamma = \int_{\Gamma^{(1)}} \underline{E}(\underline{r})\, d\underline{r} + \int_{\Gamma^{(2)}} \underline{E}(\underline{r})\, d\underline{r} = \int_{\Gamma^{(1)}} \underline{E}(\underline{r})\, d\underline{r} - \int_{\Gamma^{(1)}} \underline{E}(\underline{r})\, d\underline{r} = 0$$

__Lehet__, méghozzá azért, mert 1 db nem zárt görbe mentén végzett munka semmit nem mond el a konzervativitásról.

2.

$$V(x) = \begin{cases} \frac{1}{2}k(x+a)^2 - \frac{1}{2}ka^2, & \text{ha } x \leq 0 \ \text{I.} \\ V_0 e^{-x/\ell} - V_0, & \text{ha } x > 0 \ \text{II.} \end{cases} \qquad k, a, V_0, \ell > 0$$

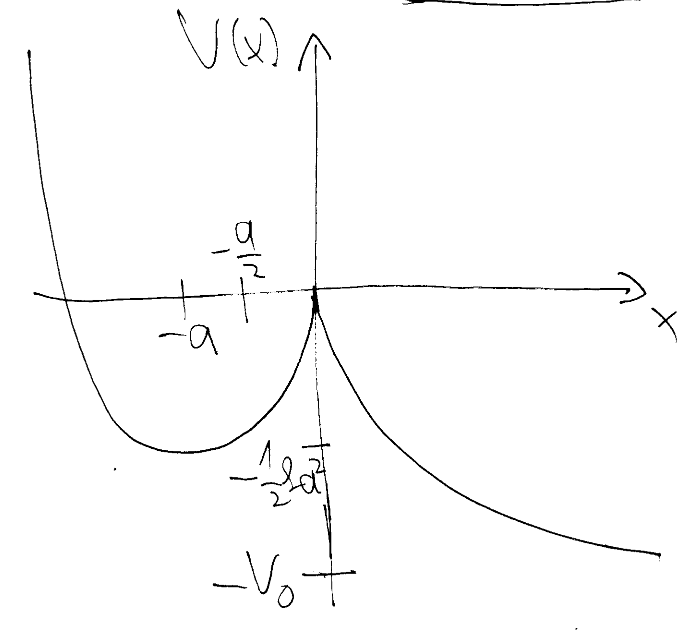
$\dot{x}\left(x = -\frac{a}{2}\right) = \frac{a}{2}\sqrt{\frac{k}{m}}$ $\longrightarrow$ $E = \frac{1}{2}m\dot{x}^2 + V(x) =$

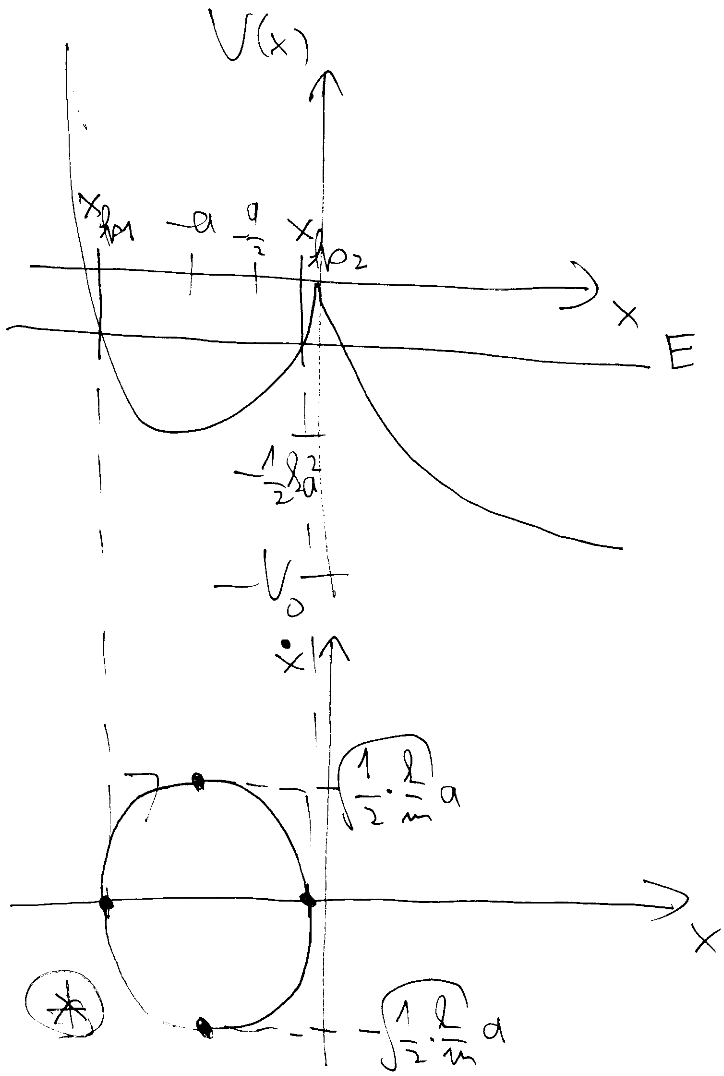
$x < 0$

$$= \frac{1}{2}m \frac{a^2}{4}\frac{k}{m} + \frac{1}{2}k\left(-\frac{a}{2}+a\right)^2 - \frac{1}{2}ka^2 =$$

$$= \frac{1}{8}ka^2 + \frac{1}{8}ka^2 - \frac{1}{2}ka^2 = -\frac{1}{4}ka^2$$

$$\Rightarrow -\frac{1}{2}ka^2 < E < 0$$





$x_{fp2} > -\frac{a}{2}$, különben nem lehetne a tömegpont az $x=-\frac{a}{2}$ pontban.

Mivel a tömegpontot az $x=-\frac{a}{2} < x_{fp2}$ helyen indítottuk, elegendő az I. tartománnyal foglalkoznunk.

• $E = V(x_{fp})$ (az I. tartományon)

$$-\frac{1}{4}ka^2 = \frac{1}{2}k(x_{fp}+a)^2 - \frac{1}{2}ka^2$$

$$\frac{1}{2}\cancel{k}a^2 = \cancel{k}(x_{fp}+a)^2$$

$$x_{fp1,2}+a = \mp\frac{a}{\sqrt{2}}$$

$$x_{fp1,2} = -a \mp \frac{a}{\sqrt{2}}$$

(Rendben van, $x_{fp1,2} \in I$.)

• $E = \frac{1}{2}m\dot{x}^2 + V(x)$

Az I. tartományon:

$$-\frac{1}{4}ka^2 = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}k(x+a)^2 - \frac{1}{2}ka^2$$

$$\underbrace{\frac{1}{4}ka^2}_{>0} = \underbrace{\frac{1}{2}m}_{>0}\dot{x}^2 + \underbrace{\frac{1}{2}k}_{>0}(x+a)^2$$

ellipszis

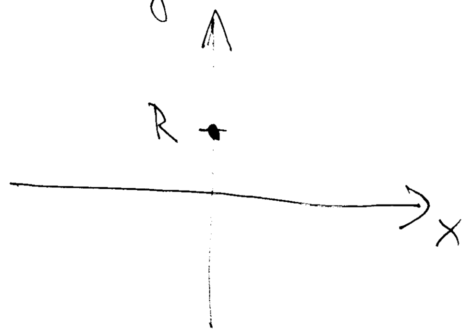
• $\dot{x}(x=-a) = \pm\sqrt{\frac{2}{m}\left(E - V(x=-a)\right)} =$

$$= \pm\sqrt{\frac{2}{m}\left(-\frac{1}{4}ka^2 + \frac{1}{2}ka^2\right)} =$$

$$= \pm\sqrt{\frac{1}{2}\cdot\frac{k}{m}}\,a$$

(*) Most csak a jelölt görbe érdekes, mert ezen van a kezdeti feltétel, így a tömegpont fázistérbeli trajektóriája ez a görbe lesz. Mivel ez zárt görbe, a mozgás periodikus.

③



$v_{fr} = \frac{1}{2} b_1 t^2$

$t=0: \ x=0, \ y=R, \ \underline{v}_{rel}=0$

$\Rightarrow r=R, \ \varphi = \frac{\pi}{2}$

$v_{rel\,r} = u_1 (1 - \tau \dot\varphi \sin\varphi)$ $\left(\begin{smallmatrix}\tau>0 \\ \text{konstans}\end{smallmatrix}\right)$

$r(\varphi) = ?$

$\beta = \beta_0 = \text{állandó!}$

$$\dot\varphi(t) = \dot\varphi(t_0) + \int_{t_0}^{t} \beta(t')\,dt' = \frac{v_{rel\,\varphi}(t_0)}{r(t_0)} + \int_{t_0}^{t} \beta(t')\,dt' = 0 + \int_0^t \beta_0\,dt' = \beta_0 t$$

$$\varphi(t) = \varphi(t_0) + \int_{t_0}^{t} \dot\varphi(t')\,dt' =$$

$$= \frac{\pi}{2} + \int_0^t \beta_0 t'\,dt' = \frac{\pi}{2} + \frac{1}{2}\beta t^2 > \frac{\pi}{2} \quad \forall t>0$$

$$\rightarrow t(\varphi) = 2\,\frac{\varphi - \pi/2}{\beta}$$

$$v_r = v_{fr} + v_{rel\,r} = \frac{1}{2} b_1 t^2 + u_1(1 - \tau\dot\varphi \sin\varphi)$$

$$r(t) = r(t_0) + \int_{t_0}^{t} v_r(t')\,dt' = R + \int_0^t \left(\frac{1}{2} b_1 t'^2 + u_1 - \tau u_1 \dot\varphi \sin\varphi\right)dt' =$$

$$= R + \frac{1}{6} b_1 t^3 + u_1 t + \tau u_1 \left[\cos\varphi(t')\right]_0^t =$$

ui. $\int_0^t \dot\varphi(t') \sin\varphi(t')\,dt' =$
$= \left[\cos\varphi(t')\right]_0^t$

$$= R + \frac{1}{6} b_1 t^3 + u_1 t + \tau u_1 \left(\cos\varphi(t) - \cos\frac{\pi}{2}\right) =$$

$$= R + \frac{1}{6} b_1 t^3 + u_1 t + \tau u_1 \cos\varphi(t)$$

$$r(\varphi) \equiv r(t(\varphi)) = R_1 + \frac{1}{6} b_1 \left(2\frac{\varphi - \pi/2}{\beta}\right)^3 + u_1 \cdot 2\frac{\varphi - \pi/2}{\beta} + \tau u_1 \cos\varphi$$

$$r\left(\varphi = \frac{3\pi}{2}\right) = R_1 + \frac{1}{6} b_1 \frac{8\pi^3}{\beta^3} + u_1 \frac{2\pi}{\beta} + 0 = \underline{\underline{R_1 + \frac{4\pi^3}{3}\frac{b_1}{\beta^3} + 2\pi\frac{u_1}{\beta}}}$$

$\Uparrow$

$\varphi(t) > \frac{\pi}{2} \quad \forall t>0$