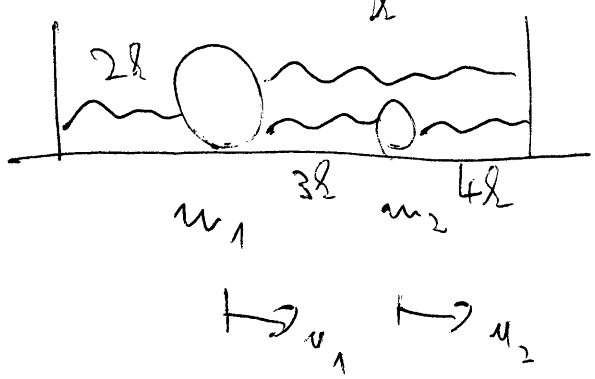
1.



$$m_1 \ddot{u}_1 = -2k u_1 - k u_1 - 3k(u_1 - u_2)$$

$$m_2 \ddot{u}_2 = -3k(u_2 - u_1) - 4k u_2$$

Probato.: $u_1(t) = A_1 e^{i\omega t}$, $u_2(t) = A_2 e^{i\omega t}$

$$\rightarrow \ddot{u}_1(t) = -\omega^2 A_1 e^{i\omega t},$$

$$\ddot{u}_2(t) = -\omega^2 A_2 e^{i\omega t}$$

$\rightarrow$

$$-\omega^2 m_1 A_1 = -6k A_1 + 3k A_2$$

$$-\omega^2 m_2 A_2 = 3k A_1 - 7k A_2$$

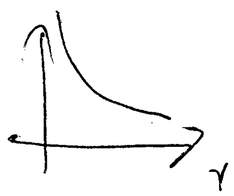
$$\begin{pmatrix} -\frac{6k}{m_1} & \frac{3k}{m_1} \\ \frac{3k}{m_2} & -\frac{7k}{m_2} \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = \underbrace{-\omega^2}_{=:\lambda} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

$$\begin{vmatrix} -\frac{6k}{m_1} - \lambda & \frac{3k}{m_1} \\ \frac{3k}{m_2} & -\frac{7k}{m_2} - \lambda \end{vmatrix} = \frac{42k^2}{m_1 m_2} + \left(\frac{6k}{m_1} + \frac{7k}{m_2}\right)\lambda + \lambda^2 - \frac{9k^2}{m_1 m_2} =$$

$$= \lambda^2 + \left(\frac{6k}{m_1} + \frac{7k}{m_2}\right)\lambda + \frac{33k^2}{m_1 m_2} \overset{!}{=} 0$$

$$\rightarrow -\omega^2_{I,II} = -\left(\frac{6k}{m_1} + \frac{7k}{m_2}\right)/2 \pm \sqrt{\left(\frac{6k}{m_1} + \frac{7k}{m_2}\right)^2 - \frac{132k^2}{m_1 m_2}} \cdot \frac{1}{2}$$

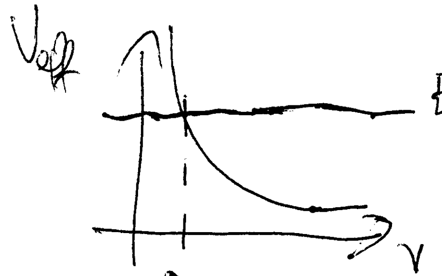
2. $V(r) = \frac{\alpha}{r^2}, \quad \alpha > 0$



$$V_{eff} = \frac{N^2}{2mr^2} + \frac{\alpha}{r^2} = \left(\frac{N^2}{2m} + \alpha\right)\cdot\frac{1}{r^2}$$

$$V_{eff}'(r) = -2\cdot\left(\frac{N^2}{2m} + \alpha\right)\cdot\frac{1}{r^3} \neq 0$$ semmilyen „r" értékre

$\Rightarrow$ NEM mozoghat körpályán.



R $\rightarrow$ Ez fordulópontja a mozgásnak!

$$\Rightarrow E = V_{eff}(r=R) = \left(\frac{N^2}{2m} + \alpha\right)\cdot\frac{1}{R^2}$$

$$\Rightarrow \dot{r}(r=R) = 0$$

$$|\underline{v}| = \sqrt{v_r^2 + v_\varphi^2} \underset{r=R}{=} \sqrt{0^2 + \left(\frac{N}{mR}\right)^2} = \frac{|N|}{mR} \overset{!}{=} u$$

$$\boxed{N = m\cdot r\cdot v_\varphi, \quad v_\varphi = \frac{N}{mr}}$$

$\Downarrow$

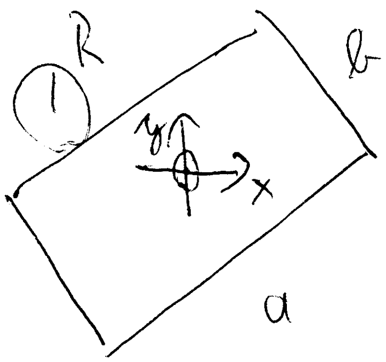
$$|N| = mRu$$

$$\rightarrow E = \left(\frac{|N|^2}{2m} + \alpha\right)\cdot\frac{1}{R^2} = \left(\frac{m^2R^2u^2}{2m} + \alpha\right)\cdot\frac{1}{R^2} =$$

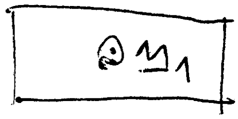
$$= \frac{1}{2}mu^2 + \frac{\alpha}{R^2}$$

(Ez természetes, hiszen $E = \frac{1}{2}m|\underline{v}|^2 + V$.)

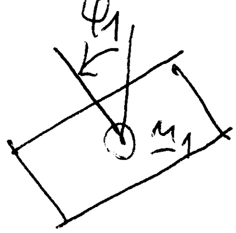
3.



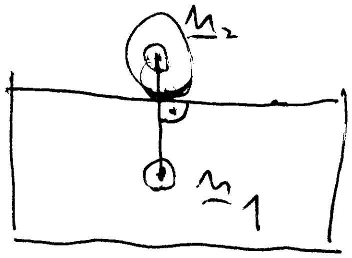
1.) $\underbrace{x_1, y_1, \varphi_1}_{\text{téglatest}}$; $\underbrace{x_2, y_2, \varphi_2}_{\text{henger}}$



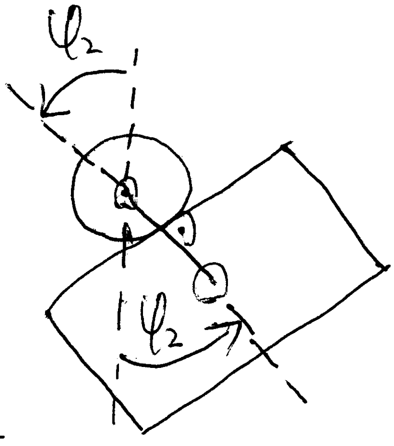
$\varphi_1 = 0$



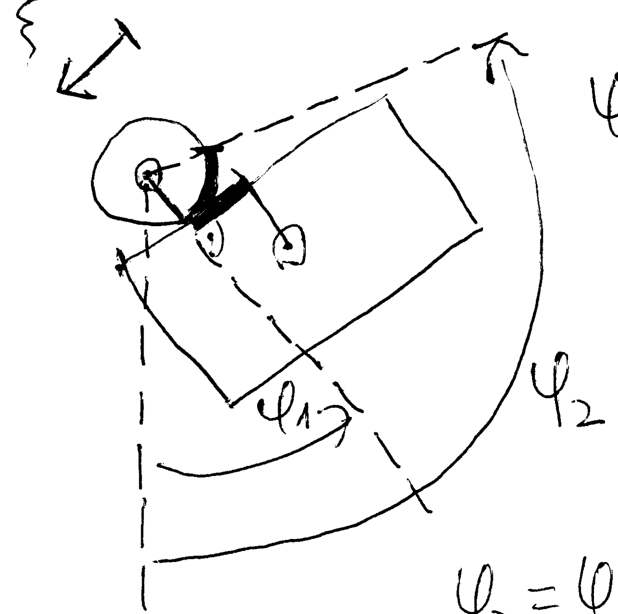
$\varphi_1 \neq 0$



$\varphi_2 = 0$



$\varphi_2 = \varphi_1$



$$\varphi_2 = \varphi_1 + \frac{\text{vastag vonal}}{R}$$

vastag vonal $= \xi$

$$\rightarrow \varphi_2 = \varphi_1 + \xi / R$$

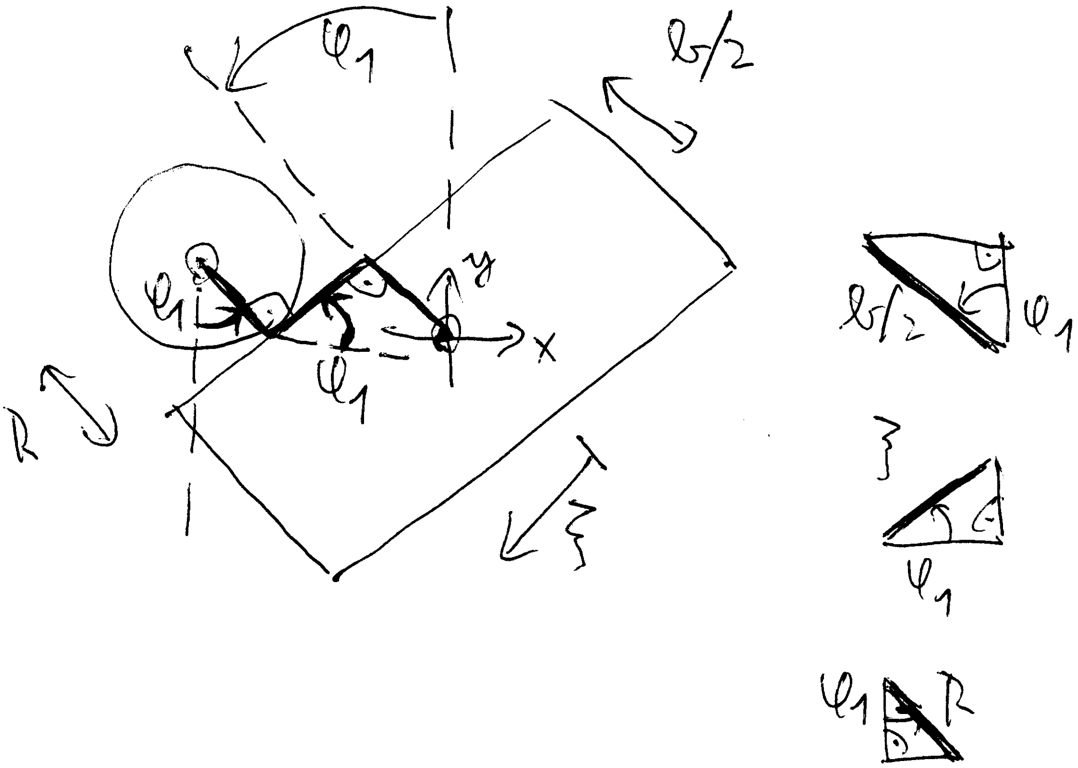
2.)
- $x_1 = 0$
- $y_1 = 0$
- „lejtő" : x_2 és y_2 nem független
- tiszta gördülés

3.) φ_1, ξ

4.) $x_1 = 0$

$y_1 = 0$

$\varphi_1 = \varphi_1$



$$x_2 = -\frac{b}{2}\sin\varphi_1 - \xi\cos\varphi_1 - R\sin\varphi_1$$

$$y_2 = \frac{b}{2}\cos\varphi_1 - \xi\sin\varphi_1 + R\cos\varphi_1$$

$$\varphi_2 = \varphi_1 + \xi/R$$

$\dot{x}_1 = 0$ $\qquad \dot{x}_2 = -\left(\frac{b}{2}+R\right)\cos\varphi_1\dot{\varphi}_1 + \xi\sin\varphi_1\dot{\varphi}_1 - \dot{\xi}\cos\varphi_1$

$\dot{y}_1 = 0$ $\qquad \dot{y}_2 = -\left(\frac{b}{2}+R\right)\sin\varphi_1\dot{\varphi}_1 - \xi\cos\varphi_1\dot{\varphi}_1 - \dot{\xi}\sin\varphi_1$

$\dot{\varphi}_1 = \dot{\varphi}_1$ $\qquad \dot{\varphi}_2 = \dot{\varphi}_1 + \dot{\xi}/R$

5.)

$$T=\frac{1}{2}m\left(\dot{x}_1^2+\dot{y}_1^2\right)+\frac{1}{2}\underbrace{\Theta_{s_1}}_{=\frac{1}{12}m(a^2+b^2)}\dot{\varphi}_1^2+$$

$$+\frac{1}{2}M\left(\dot{x}_2^2+\dot{y}_2^2\right)+\frac{1}{2}\underbrace{\Theta_{s_2}}_{=\frac{1}{2}MR^2}\dot{\varphi}_2^2=$$

$$=0+\frac{1}{24}m(a^2+b^2)\dot{\varphi}_1^2+\frac{1}{2}M\left(\dot{x}_2^2+\dot{y}_2^2\right)+\frac{1}{4}MR^2\left(\dot{\varphi}_1+\frac{\dot{\xi}^2}{R^2}\right)$$

$$V=mgy_1+Mgy_2=Mgy_2$$

$$L=T-V=\frac{1}{24}m(a^2+b^2)\dot{\varphi}_1^2+$$

$$+\frac{1}{2}M\left[\left(\left(-\frac{b}{2}+R\right)\cos\varphi_1\dot{\varphi}_1+\xi\sin\varphi_1\dot{\varphi}_1-\dot{\xi}\cos\varphi_1\right)^2+\right.$$

$$\left.+\left(\left(-\frac{b}{2}+R\right)\sin\varphi_1\dot{\varphi}_1-\xi\cos\varphi_1\dot{\varphi}_1-\dot{\xi}\sin\varphi_1\right)^2\right]+$$

$$+\frac{1}{4}MR^2\dot{\varphi}_1^2+\frac{1}{4}M\dot{\xi}^2-Mg\left(\left(\frac{b}{2}+R\right)\cos\varphi_1-\xi\sin\varphi_1\right)$$

6.) „2 általános koordináta van, ezért 2 Euler–Lagrange egyenlet lesz."