# Exoplanetary mass constraints based on topology of interacting networks

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## Motivation

**Suppose that two planets are orbiting a central star. Can we decide which one of two planets is more massive if we detect only the light emitted by the star?**

## Basic concepts & proposed work-flow

1. Observation – Transit timing variation. If an exoplanet transits it's host star along our line of sight, then a planet on an unperturbed Keplerian orbit will transit with a fixed, predictable period. When an exoplanet orbit is perturbed by interactions with other planets, the perturbed orbits can give rise to transit timing variations (TTVs). These TTVs can be used to infer the presence of unseen companions, or if the companion also transits, can be used to provide precise information on the mass of the companion planet.

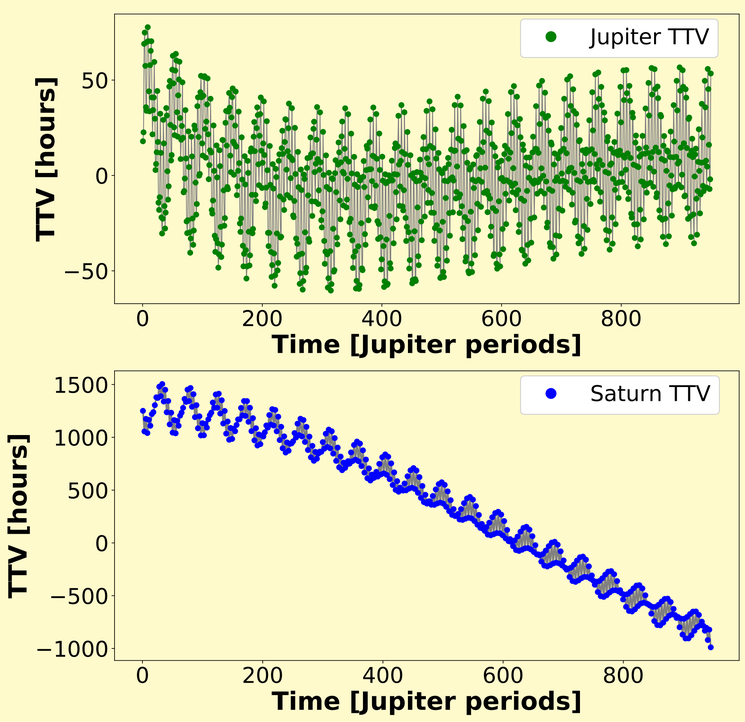
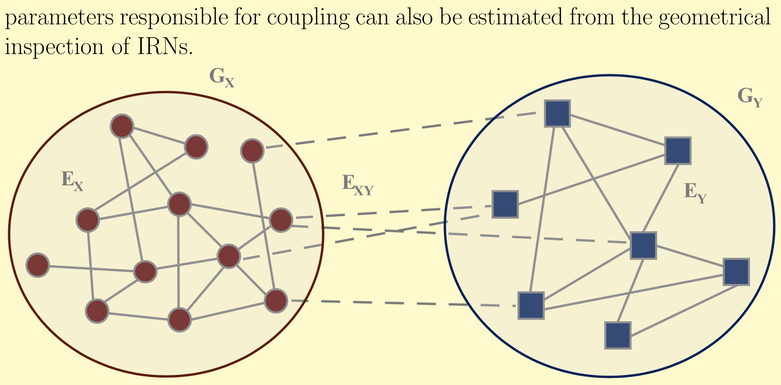



Figure 1: Orbital period variation of Jupiter and Saturn due to their mutual gravitational interaction.

2. Phase space reconstruction Observing a dynamical system often yields limited knowledge about its state space variables. Embedding theorems allows us to infer the underlying dynamics from a scalar time series once the phase space trajectory has been reconstructed from the observed signal. This is possible when the variable corresponding to the observable affects the other variables, then the variables governing the system's dynamics are coupled.

3. Coupling direction – Inter-system Recurrence network. It has been shown that the coupling direction between two entangled dynamical systems can be unfolded by the topology of inter-system recurrence networks (IRNs). Common measures of IRNs (such as *cross-edge density, cross-clustering, or cross transitivity*) based on cross-recurrence plots are capable to distinguish the driven and the driver system. Moreover, system parameters responsible for coupling can also be estimated from the geometrical inspection of IRNs.




**Coupling in gravitational three body problem**

$$m_c\mathbf{R}_c = \mathcal{G}m_c m_i\frac{\mathbf{r}_i}{r_i^3} + \mathcal{G}m_c m_j\frac{\mathbf{r}_j}{r_j^3}$$

$$m_i\mathbf{R}_i = \mathcal{G}m_i m_j\frac{\mathbf{r}_j - \mathbf{r}_i}{|r_j - r_i|^3} + \mathcal{G}m_i m_c\frac{\mathbf{r}_i}{r_i^3}$$

$$m_j\mathbf{R}_j = \mathcal{G}m_j m_i\frac{\mathbf{r}_i - \mathbf{r}_j}{|r_i - r_j|^3} + \mathcal{G}m_j m_c\frac{\mathbf{r}_j}{r_j^3}$$

## Case study – synthetic 2-planet system

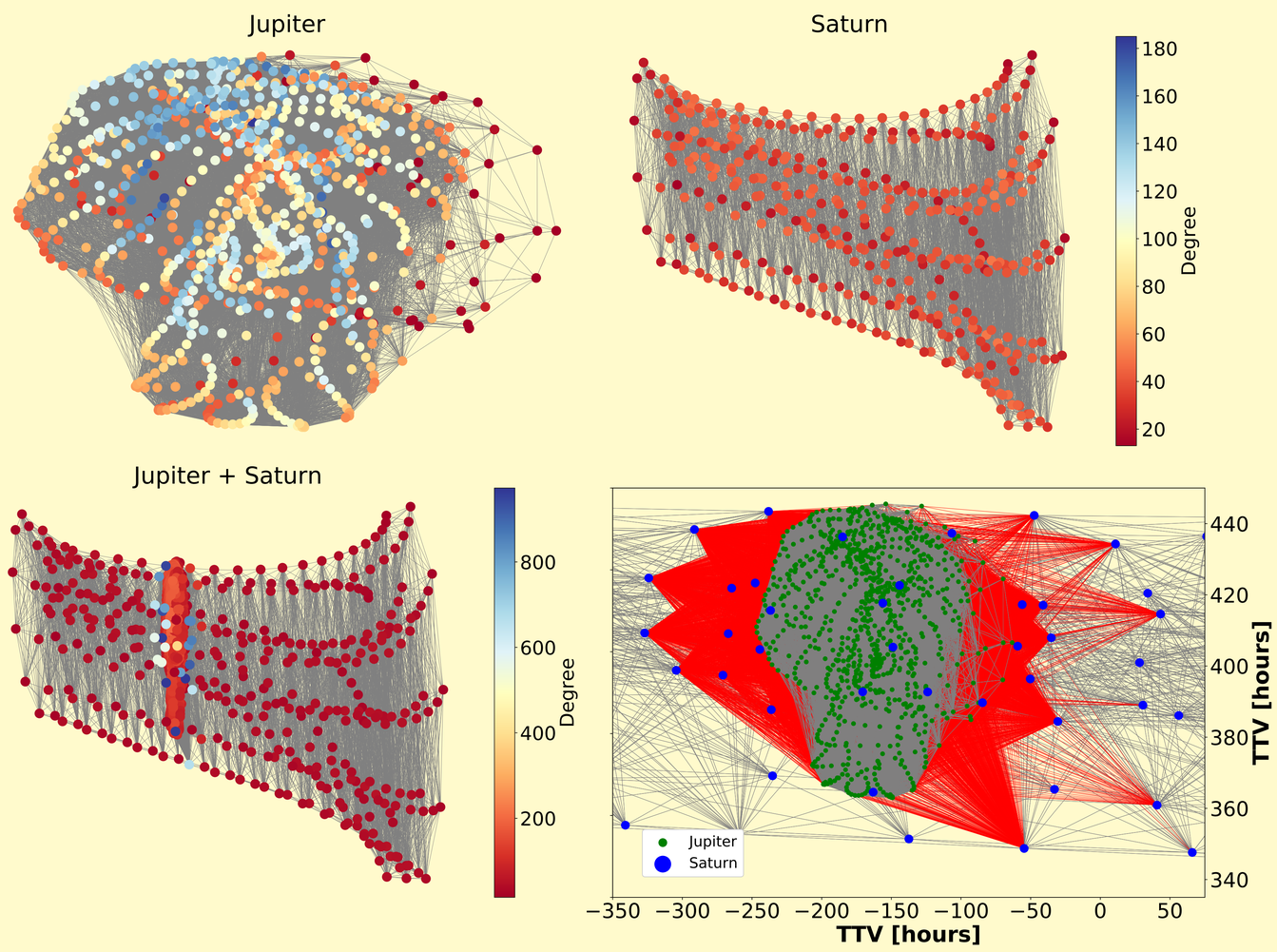



Figure 2: Recurrence networks build from the TTVs in Figure 1. Calculations were performed by PyUnicorn package.

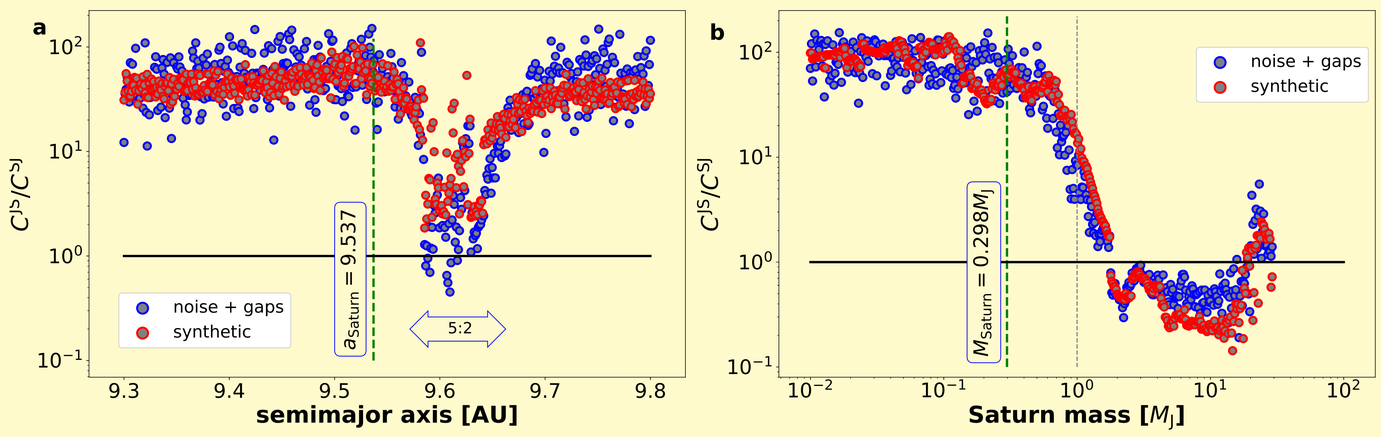



Figure 3: The ratio of cross-clustering coefficients ($\mathcal{C}^{\mathcal{XY}}$). The driver system always shows larger $\mathcal{C}^{\mathcal{XY}}$. In case of synchronization the ratio is in order of unity and the method breaks down.

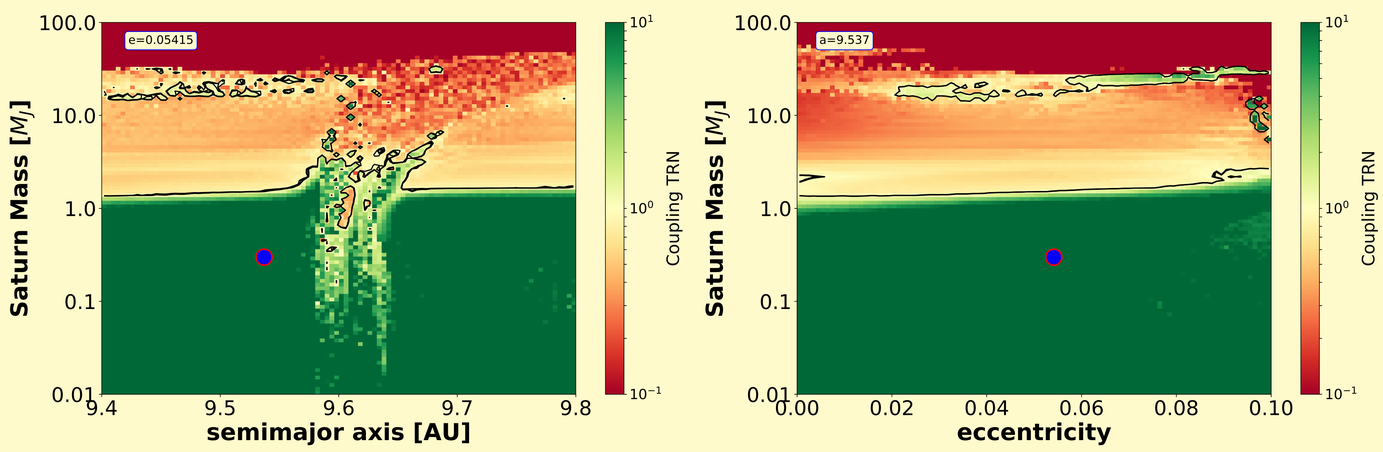



Figure 4: Parameter space exploration to certify the robustness of IRN measures. The mass relation of Jupiter and Saturn can be easily determined if they were exoplanets somewhere in the Milky Way.

## Application to real exosystems

| KOI | $C^{b\to c}/C^{c\to b}$ | $\Delta$ | IRN status | KOI | $C^{b\to c}/C^{c\to b}$ | $\Delta$ | IRN status |
|---|---|---|---|---|---|---|---|
| Kepler-23bc | 12.325 (4.13) | 0.0077 | **confirmed** | Kepler-307bc | 2.054 (1.596) | 0.005 | **confirmed** |
| Kepler-23cd | 5.604 (3.166) | 0.0156 | **confirmed** | Kepler-11bc | 0.055 (0.134) | 0.0113 | **confirmed** |
| Kepler-25bc | 7.66 (4.086) | 0.0195 | **confirmed** | Kepler-11cd | 0.496 (0.718) | 0.0451 | **confirmed** |
| Kepler-24cb | 14.468 (9.333) | 0.0095 | **confirmed** | Kepler-11de | 0.468 (0.763) | 0.0577 | **confirmed** |

**References & Download:**

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